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The influence of time delay on epidemic spreading under limited resources

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HIGHLIGHTS

- A two-layer network model considers both the time delay and the resource amount.
- The recovery probability is a time-varying variable related to the resource amount.
- Multi-phase behavior is found under the influence of time delay and resource.
- The critical threshold value of time delay is found.

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ABSTRACT

While epidemic spreading naturally involves time delay due to the order of propagation, its control depends on the resource amount to be devoted. However, it is unclear how time delay impacts epidemic spreading under limited resources. Here we analyze a simple model of epidemic spreading on a two-layer network, which considers both the time delay between the layers and the resource amount. Interestingly, we find that the delay can induce first-order, continuous and hybrid phases as well as transitions among them, depending on the resource amount, the connection strength between layers and their internal structures. In addition, we find that there is a critical threshold of time delay such that even a small resource amount can effectively control the epidemic spreading if the delay is beyond this threshold, and a large resource amount is needed otherwise. These findings would provide useful guidelines for government decisions on the control of epidemic spreading.

1. Introduction

As is well known, important contagions such as Ebola, SARS, H1N1 form serious threats to human health, so their spreading must be under control. Typically, epidemic spreading is a dynamical process, involving regulation of many factors such as the topology of networks consisting of persons to be potentially infected, the ways of the spreading and the degree of the correlation between network layers, and the resource amount to be devoted. Thus, the control of epidemic spreading is a complex system engineering, in particular needing that public health departments cooperatively make timing, correct decisions. However, the problem that we face is how epidemic spreading is controlled under limited resources. Clearly,

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understanding the influences of important factors on epidemic spreading is an important step towards making correct decisions on the effective control of epidemic spreading.

Given the complexity of epidemic spreading and the importance of its control, it is strongly needed to develop mathematical models to reveal how epidemics spread (so that government departments can make correct decisions). In fact, it has been verified that mathematical models are an important tool for analyzing the mechanisms of epidemic spreading and its control [1–3]. Until now, many mathematical models of epidemic spreading have been proposed and analyzed, e.g., the susceptible–infected–susceptible (SIS) model [4], the susceptible–infected–recovery (SIR) model [5], the susceptible–infected–recovered–susceptible (SIRS) model [6], and the susceptible–exposed–infected–recovered (SEIR) model [7]. These models have interpreted some phenomena occurring during epidemic spreading [8–11], but their limitations are also apparent. This is because the models neglected some important factors, e.g., the amount of resource used to control epidemic spreading, network layers that contagions spread from a population to another, and time delay due to the disease spreading between different populations. In the current highly-developed world, these factors become increasingly important in the control of epidemic spreading.

First, the public resource expenditure has important influence on the efficiency of disease containment. In fact, the amount of resource to be invested is directly related to the recovery rate of infected individuals. Moreover, the more amount of resource is invested, the higher recovery probability the infected individual has. Many studies (based on networks) tried to arrive at the goal of controlling an epidemic outbreak by optimizing the balance between the public health care and immunization resources, i.e., by minimizing the number of infected individuals based on an optimal allocation [12–15]. Recently, Chen et al. applied the quenched and heterogeneous mean-field theory to solve an optimization problem of how limited resources are best allocated according to the minimization prevalence [12]. Shai et al. analyzed a SIR process on coupled networks where nodes are limited to interact with a maximum number of neighbors [13], and found that in the absence of resource constraint, a positive correlation coupling leads to a lower epidemic threshold than a negative one, but in the presence of resource constraint, the spreading is less efficient in the former case than in the latter case. Similar results were also obtained in Ref. [14] wherein Chen et al. assumed that the recovery rate was positively correlated with an average resource devoted to each infected individual. They also found that only when the amount of resource was beyond a certain critical value, can a disease outbreak effectively be eradicated [15]. We note that previous studies mainly considered that epidemics spread on monoplex networks, but less considered both epidemic spreading on multilayer networks and limited resources.

Second, epidemic spreading on monoplex networks or multilayer networks naturally involves time delay due to the order of propagation. Such a delay would play an unnegligible role in the propagation process of a contagion, and is frequently used to model the exposed period of infectious diseases, the infection period of patients, or the immunity period of recovery of the disease [16–23]. Zhang et al. proposed a modified SIRS model with time delay, and found that the time delay of the assessment of epidemic risk can cause periodic outbreak of diseases [24]. Other authors proposed SIR and SIS models with time delay to investigate the impact of infection delay and propagation vector on spreading behaviors on complex networks [25–28]. Fu et al. studied a time-delayed SIS model of epidemic spreading on complex heterogeneous networks, and numerically showed that the time delay can affect the convergence speed at which the disease reaches equilibria [29]. In addition, the authors in [30] analyzed a SEIRS model of epidemic spreading on complex networks, which considered that time delay was the infection period, and obtained results on effects of the delay.

In a short, time delay and resource amount are two important factors affecting epidemic spreading on monoplex or multilayer networks, and their respective influences have been investigated. However, it seems to us that no models consider these two factors simultaneously. In particular, how time delay impacts epidemic spreading on multilayer networks under limited resources remains unexplored.

In this paper, we study a modified SIS model, which considers that a contagion spreads on a two-layer network and the resource amount is limited, and assumes that there are time delays between the layers. By model analysis, we find interesting phenomena, e.g., time delay can induce three kinds of phases (i.e., first-order, continuous and hybrid phases) and transitions among them but such transitions depend on other factors of the system such as the resource amount, the connection strength between layers and their internal structures; and there is a critical threshold of time delay such that the epidemic spreading can effectively be controlled even by a small resource amount if the delay is beyond this threshold, and the control needs a large resource amount otherwise.

2. Methods

In order to reveal the essential mechanism of how time delay affects epidemic spreading, we consider a two-layer network, where each sub-network is modeled by a set of SIS equations, and it is assumed time delays exist between the layers but no time delay exists between any two nodes in the inside of each sub-network. In both sub-networks, each node may switch between susceptible (S) and infected (I) states, and the connections between the nodes represent the network sides along which the disease can propagate. The nodes with state S can be infected simultaneously by intra-layer and inter-layer infected nodes, but the infected nodes may recover to susceptible nodes.

Let β_1 (β_2) represent the transmission probability in network A (B) that an infected node spreads the disease to a neighbor to which it shares a link, γ_1 (γ_2) the probability of transmission from a node in B (A) to a node in A (B), i.e., each of γ_1 and γ_2 represents a connection strength between the two layers, and $\mu_1(t)$ ($\mu_2(t)$) the recovery probability in network A (B) that an