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Stochastic sensitivity of systems driven by colored noise

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HIGHLIGHTS

- Impact of colored noise on nonlinear systems is considered.
- Stochastic sensitivity of equilibria is analyzed.
- Colored-noise-induced excitability in FitzHugh–Nagumo model is studied.

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ABSTRACT

We study a response of the general nonlinear dynamic system to the colored noise. A stochastic sensitivity analysis of equilibria forced by small exponentially-correlated Gaussian noise is carried out. For the stochastic sensitivity matrix of the equilibrium, a system of matrix equations is derived, and explicit solution is found in 2D case. Applying these theoretical results, we study a response of FitzHugh–Nagumo model to colored noise. We analyze how dispersion of random states near the deterministic equilibrium depends on the characteristic time of the power-limited colored noise. The effect of the colored-noise-induced excitability is discussed.

1. Introduction

It is well known that real systems operate in the presence of random perturbations of a different nature. The choice of an adequate mathematical model for acting noises is extremely important in the analysis of stochastic effects. One of the widely used models is the white noise. This noise has no characteristic time scale, is fast in comparison with the relevant physical system time scale. Such idealization of fluctuations is not exactly realized, and can result in wrong conclusions [1]. Normally, a system and noise operate together, so their time scales must be agreed. Therefore, it is important to model noise with definite characteristic time. Colored noise is one of the frequently used models where the characteristic time is connected with the decay of the correlation function, and the correlation time is the basic characteristic.

An important role of the colored noises was discovered in many physical systems, e.g. lasers [2–5], magnetic systems [6], stick–slip dynamics of mechanical systems [7], volcanic dynamics [8], seismology [9], biochemistry [10], population dynamics [11,12], microbial growth kinetics [13], tumor-immune dynamics [14].

There are many studies where a specific role of colored noises was analyzed in such phenomena as noise-induced transitions [15–17], stochastic [18,19] and coherence resonance [20], stochastic bifurcations [21], and order-chaos transitions [22,23]. An influence of colored noise on the oscillators was studied in [24,25].

Colored noises are frequently modeled by the solutions of linear stochastic systems with white uncorrelated Gaussian noises. As a result, an analysis of the influence of colored noises can be reduced to the study of extended dynamic systems forced by white noises. Stochastic dynamics of such extended systems can be formally described by Fokker–Planck equation.

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However, a mathematical analysis of this equation in multidimensional case is extremely laborious. For systems forced by white noise, various asymptotic methods are widely used [26]. In [27–29], a constructive approach based on the stochastic sensitivity function technique has been elaborated and applied to the analysis of the various white-noise-induced phenomena.

The aim of this paper is to extend this approach to the case of general nonlinear dynamic systems forced by colored noise.

In Section 2, the stochastic sensitivity analysis of equilibria for multidimensional dynamic systems with exponentially-correlated Gaussian normalized vector noise is carried out. For the stochastic sensitivity matrix of the equilibrium, the system of matrix equations is derived.

The solution of this equation for two-dimensional dynamic systems is presented in Section 3. Here, explicit formulas connecting elements of the stochastic sensitivity matrix with parameters of the deterministic system and correlation time of the colored noise are found.

In Section 4, these theoretical results are applied to the study of FitzHugh–Nagumo model forced by colored noise. A dependence of the dispersions of random states near deterministic equilibrium on the characteristic time of colored noise is analyzed, and the effect of the colored-noise-induced excitability is discussed.

2. Stochastic sensitivity of equilibrium in system forced by colored noises

Consider a nonlinear dynamic system

$$\dot{x} = f(x, r), \tag{1}$$

where x is an n-vector, $f(x, r)$ is a sufficiently smooth vector-function, r is an m-vector of random disturbances. It is assumed that the unforced system ($r = 0$) has an exponentially stable equilibrium $\bar{x}:\ f(\bar{x}, 0) = 0$.

We will study how this equilibrium is influenced by the small colored noise. In this paper, random disturbances $r(t) = \varepsilon s(t),\ s = (s_1, \dots, s_m)^\top$ of the noise intensity ε are modeled by the following Langevin equations:

$$\dot{s}_i = -a_i s_i + \sigma_i\sqrt{2a_i}\xi_i(t), \quad a_i > 0. \tag{2}$$

Here, $\xi_i(t)$ $(i = 1, \dots, m)$ are standard uncorrelated Gaussian white noises with parameters $\langle \xi_i(t) \rangle = 0,\ \langle \xi_i(t)\xi_i(t') \rangle = \delta(t - t')$. The system (2) generates uncorrelated colored noises with parameters

$$\langle s_i(t) \rangle = 0,\ \langle s_i(t)s_i(t') \rangle = \sigma_i^2 \exp\left(-a_i|t - t'|\right), \quad \tau_i = \frac{1}{a_i},$$

where parameters a_i define correlation times τ_i of these colored noises.

Let $x^\varepsilon(t)$ be a solution of the stochastic dynamic system

$$\dot{x} = f(x, \varepsilon s) \tag{3}$$

forced by colored noises formed by (2). To study a dispersion of $x^\varepsilon(t)$ near the equilibrium $\bar{x}$ for small colored noise, we will use the following asymptotics:

$$y(t) = \lim_{\varepsilon \to 0} \frac{x^\varepsilon(t) - \bar{x}}{\varepsilon}.$$

Dynamics of the pair $y(t), s(t)$ is governed by the stochastic linear system:

$$\dot{y} = Fy + Gs \tag{4}$$

$$\dot{s} = -As + C\xi(t). \tag{5}$$

Here

$$F = \frac{\partial f}{\partial x}(\bar{x}, 0), \qquad G = \frac{\partial f}{\partial r}(\bar{x}, 0),$$

$$A = \begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_m \end{bmatrix}, \quad C = \begin{bmatrix} \sigma_1\sqrt{2a_1} & & 0 \\ & \ddots & \\ 0 & & \sigma_m\sqrt{2a_m} \end{bmatrix}, \quad \xi = \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_m \end{bmatrix}.$$

For the extended $(n + m)$-dimensional vector $z = \begin{bmatrix} y \\ s \end{bmatrix}$, one can write the system

$$\dot{z} = \Phi z + S\xi(t), \tag{6}$$

where

$$\Phi = \begin{bmatrix} F & G \\ O & -A \end{bmatrix}, \quad S = \begin{bmatrix} O \\ C \end{bmatrix},$$

and O is a zero matrix of the appropriate dimension.