Physica A 509 (2018) 1087–1094

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Effect of localized loading on failure threshold of fiber bundles

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HIGHLIGHTS

- A set of elements – fibers – with individual failure thresholds are considered.
- Global strength varies non-monotonically with fraction of fibers initially loaded.
- The critical fluctuation near the weakest point is different from any other point.
- It serves as a simple model for disordered interconnections, like power grids.

ARTICLE INFO

Article history:
Received 5 March 2018
Received in revised form 19 June 2018
Available online xxxx

Keywords:
Fracture
Fiber bundle model
Critical fluctuation

ABSTRACT

We investigate the global failure threshold of an interconnected set of elements, when a finite fraction of the elements initially share an externally applied load. The study is done under the framework of random fiber bundle model, where the fibers are linear elastic objects attached between two plates. The failure threshold of the system varies non-monotonically with the fraction of the system on which the load is applied initially, provided the load sharing mechanism following a local failure is sufficiently wide. In this case, there exists a finite value for the initial loading fraction, for which the damage on the system will be maximum, or in other words the global failure threshold will be minimum for a finite value of the initial loading fraction. This particular value of initial loading fraction, however, goes to zero when the load sharing is sufficiently local. Such crossover behavior, seen for both one and two dimensional versions of the model, can give very useful information about stability of interconnected systems with random failure thresholds.

1. Introduction

An interconnected set of elements sharing a load is a common situation arising in diverse contexts such as the disordered solids under stress [1–6], grids carrying current [7,8], network of computers sharing a task, network of roads carrying traffic [9,10] etc. The catastrophic failure point in such systems, while mostly an undesirable situation, is a crucial factor in fixing the operating points and thereby limiting the resources and functionality of the relevant systems. A set of elements, or fibers, having random failure thresholds and fixed between two rigid plates, is a prototype model to study the breakdown properties of a broad category of systems such as these, under simplifying yet informative assumptions.

The so called fiber bundle model was introduced in the textile industry [11] to model the strength distributions of cloths. Since then it has found wide spread applications in systems with varying degree of complexities that still has the underlying basic dynamics of threshold activated breakdown [12]. While the individual fibers are often assumed to have a linear stress–strain relation with an irreversible breakdown beyond a threshold, the overall response of the system is non-

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https://doi.org/10.1016/j.physa.2018.06.112
0378-4371/

linear. Depending upon the distribution function of the individual failure thresholds and the range of load redistribution following a local failure, the system can show nucleation driven extreme statistics to random percolative failure through avalanche dynamics [13–16]. The system size dependence of these response statistics, limiting cases of very strong or weak disorders in the system and the range of load sharing etc. are some of the important questions that are still being actively investigated [16–18].

In this work, however, we look back at the prediction of failure threshold under a constant total load, the original question of the model. In most studies, the application of the initial load in fiber bundles is uniform. Under that condition, the catastrophic failure threshold and its system size dependence are well studied and understood [19,20]. However, much less attention was paid to the systems where the loading may not be uniform at the outset (see e.g. [21]). This is, however, a very common situation that can arise in all the examples mentioned above. For example, going back to the origin of the model, a constant load can either be applied uniformly on the lower plate supported by the fibers or can be distributed between a series of loading points. In power grids the loading is known to be non-uniform (see e.g. [22]), similar situation is true for traffics on the road, computer network with redundancies and so on. Such non-uniformity of the initial loading can have very significant effect on the global failure threshold of the system. Here we show how the failure threshold, or the overall load carrying capacity of the system, varies with the fraction of system where the load is initially applied. Interestingly, we find the variation to be non-monotonic. This implies that in case of partial loading, certain fractions are to be avoided if the goal is to increase the overall failure thresholds. In other cases, where fracturing is desirable (e.g. hydraulic fracture in oil extraction), such fractions are to be targeted to achieve maximum fracture.

In the following, we first investigate the fiber bundle model in the mean field limit, where the initial loads are applied to a finite fraction of the system. Even in this simple limit, we see that given a fixed total load, the damage on the system is maximum when a finite fraction (between $1/N$, N being the system size, and 1) of it bears the initial load. In other words, the failure threshold is minimum for that fraction. This point of minimum threshold is special and has different scaling of the fluctuation of the critical load than in other points both above and below this fraction. The mean field limit has some analytical tractability and hence can bring some insights to the dynamics. We then go over to the more realistic situations of having a finite compliance of the bottom plate [23], or in other words a power law load sharing in the system [24,25], following a local breakdown. This is done for both the one dimensional and two dimensional versions of the model. In both cases, for sufficiently wide load redistribution rule, we recover the non-monotonicity of the damage fraction (and of the critical threshold), which is retained up to certain degree of load redistribution range. For very localized load redistribution, however, this non monotonicity vanishes and failure threshold becomes a monotonically increasing function of the initially loaded fraction.

2. Model

In its original form, the fiber bundle model is viewed as a set of N fibers fixed between two plates. The plates are either pulled apart (strain controlled dynamics) or a fixed load is attached to the bottom plate (stress controlled dynamics). The individual fibers are linear elastic and each of them can fail irreversibly once their failure thresholds are exceeded. The failure thresholds σ_{th}^i are drawn randomly from some probability distribution. Once a fiber fails, its share of load is then redistributed among the remaining intact fibers. The collective behavior of the system is surprisingly rich. It depends mainly on the properties of the distribution function the failure thresholds are chosen from and the way in which the load is shared between the remaining intact fibers. In this work, we will use a uniform threshold distribution between [0:1]. The load sharing mechanism will be varied. Specifically, we will study the mean field limit, where the load of a failed fiber is shared equally between all remaining intact fibers, and also the power law load sharing, where a fiber at site r will have a load share proportional to $1/|r-r'|^{\alpha}$ when a fiber at site r' fails. Of course, the limit $\alpha \to 0$ is the mean field limit and $\alpha \to \infty$ is completely local load sharing limit. In practice, depending on the spatial dimension of the problem, which is either one or two here, there is a crossover value α^* for which the behavior of the model crosses over from mean field to local load sharing limit.

The crucial difference in this work is that the load is initially applied to p fraction of the fibers. The subsequent dynamics in the simulation is the same as is usually followed in fiber bundle models i.e. the fibers having failure thresholds below the applied load on it are broken, the load carried by those fibers are then redistributed on the remaining surviving fibers depending on their distance from the broken fibers in case of power law load redistributions mentioned above or uniformly on all the fibers in case of mean field. The redistribution can trigger further breaking of the fibers and so on. For a given total load on the whole system, the system can either be stable in a state where all the surviving fibers carry a load below their respective thresholds or all the fibers are broken. The critical load for which the system just survives complete breakdown, is the critical point of the system. The simple modification of applying the load to a finite fraction of fibers initially, however, has a profound effect on the dynamics, stability and critical fluctuations of the system, which we shall investigate in the following for the mean field and both one and two dimensional cases with power law load sharing.

3. Results

3.1. Mean field model

We begin with the simplest case of the mean field version of the model, where following the failure of one fiber, the load carried by that fiber is uniformly redistributed among all the other remaining fibers. Now, if we assume p fraction of