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The public goods game on scale-free networks with heterogeneous investment

Hanchen Wang [a], Yichun Sun [c], Lei Zheng [a], Wenbo Du [a,*], Yumeng Li [a,b,*]

[a] School of Electronic and Information Engineering, Beihang University, Beijing 100191, PR China
[b] Shen Yuan Honors College, Beihang University, Beijing 100191, PR China
[c] School of Automation, Beijing Institute of Technology, Beijing 100081, PR China

HIGHLIGHTS

- Heterogeneous investments promote cooperation on scale-free networks.
- Investigate the mean payoffs of nodes to confirm that the strategy utilized by hubs is more prominent.
- Microscopic dynamical processes behind the promotion of cooperation are identified and explained.

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ABSTRACT

Heterogeneity has received increasing attention over the past decade, and it is widely considered to have a great effect on promoting cooperation. In our paper, we investigate the influence of heterogeneous investment on the cooperative behavior of the system, which is based on the number of players in different groups in the evolutionary public goods game (PGG). A parameter α is used to tune the heterogeneous investment mechanism: if $\alpha > 0$, cooperators invest more in the groups centered on high-degree nodes, but if $\alpha < 0$, the groups centered on small-degree nodes attract more investment from cooperators. Simulation results and analysis suggest that the cooperation level is enhanced in a large range. For a smaller enhancement factor r in the PGG model, the cooperation frequency decreases monotonously with the increase of α. For a larger r, the highest cooperation level can only be obtained by $\alpha = 0$, which is a non-monotonous phenomenon. By investigating the mean payoffs of nodes of different degrees for different α, it is found that the strategy utilized by hubs is more prominent in the system. Some explanations are provided for the strategy orientation of hubs and show that the investment propensity will change it. Our work may be useful for understanding the influence of individual investment in different groups on the level of cooperation.

1. Introduction

In natural, economic and social systems, cooperation is omnipresent and plays a significant role [1]. However, understanding the emergence and maintenance of cooperation in a variety of realistic systems consisting of selfish individuals is a central problem that contradicts Darwin's theory [2]. To date, scientists in many different fields have focused on game theory, which provides a powerful mathematical framework for characterizing and investigating the cooperative dilemma. The Prisoners Dilemma Game (PDG) [3] and the Snowdrift Game (SG) [4], two simple games, provide a typical paradigm

* Corresponding authors.
E-mail addresses: wenbodu@buaa.edu.cn (W. Du), liyumeng@buaa.edu.cn (Y. Li).

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to explain cooperative or uncooperative behavior through pairwise interactions. To provide a reasonable explanation for group interactions beyond pairwise interactions, the public goods game (PGG) has been proposed as a more general model for investigating global warming, public resources, team cooperation and social species problems [5].

In the typical PGG model [6,7], all N players must independently decide whether to invest into a public pool (C) or not (D). The total amount of the PGG group will be multiplied by an enhancement factor r ($r > 1$) first and then averaged to all players, regardless of their selections in the group. According to a theoretical prediction, the dominating strategy is represented by the defection causing the emergence of dilemma, which is similar to the PDG. For all players, becoming "free-riders" (D) is the perfect strategy in the Nash equilibrium because each player will do better by contributing nothing than when they contribute something, regardless of what the other players do. This is apparently contradictory to the widely observed cooperative behaviors [8].

A seminal work by Nowak and May found that simple spatial structure can induce the emergence and persistence of cooperation [9]. Along with this finding, many studies focused on the interplay between evolutionary games and the various spatial topologies formed by local interactions among the structured population, such as games on regular lattices [10–14,7,15], small-world networks [16–20] and scale-free networks [21–25]. The evolutionary cooperation of public goods game has been widely studied over the past few years. Quite a few mechanisms have been proposed such as the punishment [26,27], reproductive ability [28], and group formation [29], which play an important role in the evolution of cooperation. Hauert and Szabó et al. introduced the voluntary participation mechanism into the PGG, which resulted in a substantial and persistent willingness to cooperate [30,7]. Szolnoki et al. firstly raised the effect of players' inhomogeneous activity and launched a series of related outstanding work [31–33]. Then, Guan et al. studied the effects that both players' inhomogeneous activity and noise on cooperation in PGG with regular spatial structure and found that the persistence of cooperation can be markedly enhanced [34]. Perc et al. introduced social diversity of different distributions and found that the power-law distribution enables the best promotion of cooperation [35]. Santos et al. studied social diversity by means of heterogeneous graphs, which can sharply promote the emergence of cooperation in PGG [6].

Recently, the introduction of heterogeneity has proved to be a great promoter of cooperation in the development of evolutionary theory. To date, the following aspects of heterogeneity have been the main research focus: the heterogeneity of personality [35,36], strategy updating rules [37], enhancement factor [38], investment [39,40] and payoff allocation [41]. The player's degree, history payoff and cooperative qualities are utilized to make the investment assigned to each PGG group heterogeneous. Cao and Lei et al. studied an unequal investment mechanism on scale-free networks in which the investment of players is related to its degree [42,43]. Yuan et al. investigated a heterogeneous investment mechanism that is based on the fraction of cooperators inside a PGG on a square lattice [44]. It is clear that the heterogeneous investment mechanism can promote cooperation; however, each cooperator has a different total investment. To ensure that the total investment of all cooperators is the same, a new model is proposed to more clearly explore the effect of heterogeneous investment on cooperation. Our simulation results and analysis reveal that heterogeneity is still beneficial to cooperation under group interactions. Compared with the qualitative analysis from the aspect of principle in the previous works, we demonstrated some deeper explorations of the heterogeneous investment. From the mean payoffs of nodes, it can be found that hubs play a prominent role in the system and the theoretical approximation is provided by the mean-field calculation. Moreover, the microscopic dynamical processes for this are also identified and explained. Our work may be useful to gain a deeper understanding about the influence of individual investment.

The paper is organized as follows. In Section 2, we describe models of evolutionary games and the strategy-updating rule used in this work. Simulation results and theoretical analyses are introduced in Section 3. The paper is concluded in Section 4.

2. The model

The present work is followed absolute payoff version in Ref. [6] which discussed two versions of PGG and the other is the degree normalized version. Each individual x participates in $(k_x + 1)$ PGG groups (k_x is the degree of individual x), centered on x and its neighbors. Each PGG group is composed of a central individual and its neighbors. In the standard PGG game, cooperators assign their total assets ($c = 1$) equally to every PGG group in which they participate, and the defectors do not. Without loss of generality, the total investment value of the cooperators is the same as the standard PGG ($c = 1$). The investment of the individual x given to the PGG group whose center is the individual y is given by

$$c_x^y = \frac{s_x(k_y + 1)^\alpha}{\sum_{j\in\Omega_x}(k_j + 1)^\alpha} \tag{1}$$

where k_y is the number of neighbors for individual y, Ω_x is the group of individual x and all of x's neighbors and s_x represents the strategy of x (1 if x is a cooperator, 0 if x is a defector). The total assets are multiplied by an enhancement factor r and are redistributed uniformly among all players in the group. Based on the mechanism above, the payoff for individual x that is obtained from the PGG group formed by individual y is expressed as

$$h_{x,y} = \frac{r}{k_y + 1}\sum_{i=0}^{k_y} c_i^y - c_x^y \tag{2}$$