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Critical phenomena of spreading dynamics on complex networks with diverse activity of nodes

Li-xin Zhou [a], Jie Lin [a,*], Yu-qing Wang [b], Yan-feng Li [a], Run-sheng Miao [a]

[a] *School of Economics and Management, Tongji University, Shanghai 200092, China*
[b] *Business School, University of Shanghai for Science and Technology, Shanghai 200093, China*

HIGHLIGHTS

- A model of spreading dynamics in complex networks with different nodes activity rate is proposed.
- Different activity probability of individuals correlates with its degree in homogeneous and heterogeneous networks.
- Critical threshold of spreading dynamic can be explained by the eigenvalues and eigenvectors of the correlation matrix.
- The basic reproductive number of spreading dynamic depends on both infection rate and activity rate.

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ABSTRACT

In this paper, we propose a new model to investigate the spreading dynamic and critical phenomena on complex networks based on SIR model. Different from previous studies, we combine the effects of activity rate and infected rate on spreading process. Network nodes become active according to different probability correlated with its degree. Active infected nodes can interact all active susceptible neighbors, meanwhile, recover at a certain probability. By means of the mean-field equations, we find the basic reproductive number and critical threshold of spreading dynamic can be explained by the eigenvalues and eigenvectors of the correlation matrix. Furthermore, we utilize analytical and numerical simulations to explore the critical phenomenon and spreading dynamics of homogeneous and heterogeneous networks respectively. Our results indicate that both homogeneous networks and heterogeneous networks of the model exhibit a critical threshold consists of critical activity rate and infection rate in the spreading dynamic. The critical threshold of infection rate is increased by node activity, and node activity also shows a critical phenomenon given certain infection rate. Results validate that our model is a feasible and economical method to control spreading dynamics and promote further application of innovation diffusion, viral marketing in reality.

1. Introduction

Spreading dynamics processes can explain many important activities well and play significant roles in real world, such as malicious software on the Internet, rumors in the social life, cascading failure in electronic circuits, virus spreading in the crowd, etc. Thoroughly comprehension of them has important significance for strategies and policy makers to control and make use of spreading dynamics in diverse fields. Moreover, many social, physical and biological systems can be well described by complex network, where nodes represent typical units and links represent interactions between pairs of units.

* Corresponding author.
E-mail address: jielinfd@163.com (J. Lin).

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In recent years, it has been proved to be a valuable tool for describing of behavior and interaction between individuals. Therefore, complex networks have attracted many scientists attention on the field of spreading phenomenon [1–7].

Modeling of propagation phenomena on contact networks have attracted a great deal of interest due to its practical significance. Peculiar topological properties of complex networks characterize the dynamics propagation process well, especially the Small-World (SW) property and Scale-Free (SF) connectivity distributions appear as common features of many real-world networks. Furthermore, researchers investigate the spreading dynamics of complex networks based on the SIS (Susceptible–Infected–Susceptible) and SIR (Susceptible–Infected–Removed) model, which susceptible individuals become infected by contact with the infected, and infected individuals can become healthy again or die. Furthermore, Pastor propose mathematical equations to study propagation thresholds on special-case networks (such as Erdős–Rényi networks, BA networks and homogeneous networks) and find the threshold tend to zero for infinite power-law networks [8–11]. In other words, the structure of complex networks plays a critical role in the spreading dynamics processes. Meanwhile, node correlations are very important in determining the physical properties of these networks [12,13].

In addition, Callaway observed the different correlations of network topology have important effects in the percolation transition and spreading dynamics [14]. So most recently, researchers study the spreading dynamics process in homogeneous networks and heterogeneous networks respectively [15–17]. However, social networks are intrinsically dynamical entities. In a dynamic social setting, people are likely to respond by social distancing or quarantine changes in behavior that are perceived to reduce the likelihood of infection [18]. Such behavioral adaptations will change the network topology and feed back into the dynamics of epidemic spreading [19]. Besides, Ghoshal and Kan explain the Multiplex networks, which compose of several lawyers of networks interrelated with same individuals, propose a nature way to investigate spreading dynamics in different categories and different context [20,21].

Research shows that individual activity and behavior will significantly impact the spreading dynamics [22–24]. Although those above models describe the instantaneous and fluctuating dynamics of many networks well, those models cannot define the activity of each individual reasonably. Researchers constructed the time varying networks with activity-driven nodes, which assume that individual will be connected by an active one with equal probability [25–28]. In other words, active nodes have same time interval and they active once during each time step. Liu present that individuals' behavior and frequency in real world cannot be explained by Poisson distribution, and propose a simple model with active nodes based SIR model, which to explore the spreading dynamics phenomenon in complex networks [1]. Consistent with Rizzo's views [29], they suppose that individuals have the same activity probability to became active in the networks. Furthermore, Perra reshapes the binomial distribution model to describe the node's different activities [30]. Although above models consider the activity of nodes, they suppose that all individuals have same activity probability. Rather, the activity rate of a real-world individuals always correlate with itself state and size of neighbors group, exist obvious difference between homogeneous networks and heterogeneous networks.

In this paper, we investigate the spreading dynamic in homogeneous networks and heterogeneous networks with activity nodes. Different with previous models, we extended the SIR model to explore the spreading dynamic on complex networks with matrix eigenvalue and eigenvector, which explains the spread phenomenon of the networks well. During each time step in our model, each active infected node affects with all its susceptible neighbors, while an activity susceptible node can be infected by its activity infectivity neighbors or it will never be infected. By means of mean-field theory to analyze the spreading dynamic processes on homogeneous networks and heterogeneous networks. Then we utilize approximate analytical and numerical simulations to validate the epidemic threshold, the basic reproductive number and the final steady-state of the model on ER and BA networks.

The rest of this paper is organized as follows: we propose a new spreading dynamic model of complex networks with activity nodes in Section 2. In Section 3, we utilize mean-field equations to investigate the spreading dynamic processes with the eigenvalues and eigenvectors, and explore the impact of node activity on critical threshold and spreading scale of homogeneous and heterogeneous networks In Section 4, numerical simulation is investigated to validate the epidemic threshold and steady-state on different networks. Conclusions and discussions are given in Section 5.

2. Model

In this section, we propose a simple model that combines the activity rate and infection rate based on SIR model, to investigate the complex process of spreading dynamic contains underlying propagation phenomena, individuals' behavior and interaction between them. During the social network, we consider that if the individuals have more neighbors and links, which meaning those individuals are more active than others. Therefore, we give each individual different activity rate based on the network structure, analyze the effects of activity rate of individuals in the spreading process. Hence, we consider a population of N individuals and epidemic spread along the quenched network $A = (a_{ij})_{N\times N}$, $a_{ij} = 1$ if there is a link connecting node i to node j and 0 otherwise. Contacts between those nodes are governed by the following set of rules.

- Each node of the network has three states: S (susceptible), I (infected), R (removed);
- An arbitrary node i becomes *active* with probability μ_i and *inactive* with probability $1 - \mu_i$, μ_i is proportional related to degree of node i. The dynamic process is random and hidden Markov processes (HMP) in the sense that nodes have no memory of the previous timesteps;