Physica A 509 (2018) 479–491

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Target-encirclement control of fractional-order multi-agent systems with a leader ☆

Lipo Mo [a,*], Xiaolin Yuan [a], Yongguang Yu [b]

[a] School of Science, Beijing Technology and Business University, Beijing 100048, PR China
[b] Department of Mathematics, Beijing Jiaotong University, Beijing 100044, PR China

HIGHLIGHTS

- The fractional-order multi-agent systems with a leader is considered.
- The target-encirclement control problem is investigated.
- A novel target-encirclement control protocol is proposed.
- Some sufficient conditions for target-encirclement are given.

ARTICLE INFO

Article history:
Received 13 February 2018
Received in revised form 10 May 2018
Available online xxxx

Keywords:
Fractional-order multi-agent systems
Target-encirclement control
Tracking
Leader

ABSTRACT

This paper addresses the target-encirclement control problem of fractional-order multi-agent systems(FOMASs) with a leader, where the state information of the target can only be obtained by the leader while followers cannot. First, a new target-encirclement control protocol is proposed, where the state estimators of the leader and the target are induced. Next, some sufficient conditions based on the theory of Mittag-Leffler stability are given for achieving target-encirclement control of FOMASs with a leader. Finally, simulation results are provided to demonstrate the effectiveness of the presented results.

1. Introduction

Over the past few years, the distributed coordination control problem of multi-agent systems has drawn more and more attention from researchers because of its wide applications in biology, control engineering and so on. Particularly, as one of the basic issues of the distributed coordination control, the consensus problem has been investigated extensively [1–7]. In modern military, it is often necessary to make all agents encircle some targets so that attack and reconnoiter the situations of the enemy. The target-encirclement control problems have been a key point in researches in view of their potential applications in the civilian and military communities, such as automated Visual Surveillance [8], formations of mobile robots [9], protection of unmanned ground vehicles [10]. Therefore, it is of interest to study the target-encirclement control problem of multi-agent systems in a distributed manner. For example, in [11] the finite-time rotating encirclement control of multi-agent systems was investigated and in [12] a new protocol was proposed to make all agents enclose and

☆ This work is supported by National Natural Science Foundation of China (Grant No. 61304155 and 11371049).
* Corresponding author.
E-mail addresses: beihangmlp@126.com (L. Mo), 1171681540@qq.com (X. Yuan), ygyu@bjtu.edu.cn (Y. Yu).

https://doi.org/10.1016/j.physa.2018.06.014
0378-4371/

track targets in the same configuration as that of the targets asymptotically, where the same configuration means that the formation of the targets is similar to the formation of the agents, and the agents are surrounding the targets.

The above papers discussed the dynamical behavior of multi-agent systems with integer-order integrator. However, due to the fact that the fractional calculus has the trait of memory and heredity, the system models could be depicted by fractional operators better. And the memory feature of fractional calculus has been used widely [13,14]. Besides nowadays, it has been studied in various fields by searchers, such as information communication [15,16] and engineering control [17,18]. Especially in recent years, considering the memory property of fractional calculus, it is meaningful to develop the potential application in consensus control of FOMASs and numerical good research results have been reported [19–23]. For integer-order multi-agent systems without leaders, the protocols proposed in [11] could enforce multi-agent systems to achieve finite-time rotating encirclement control, however, their results cannot be extended to FOMASs directly due to the fact that there is no periodic solution and finite-time equilibrium for fractional-order equation [24]. Hence, we propose a new control law to deal with these difficulties and we get the sufficient conditions for achieving target-encirclement control asymptotically with no rotation.

Motivated by the above discussions, in this paper, we investigate the target-encirclement control problem of FOMASs with a leader, where the state information of the target can only be obtained by the leader, while followers cannot. In military campaign, the target, the leader and followers could be seen as the enemy plane, the scout plane and fighter planes respectively. In actual combat, only the scout plane has detection ability, it could detect the state and position information of the enemy plane actually, while fighter planes cannot. Therefore, to make sure the target-encirclement control could be achieved, observers are designed for all fighter planes to observe the scout plane's state, besides, estimators are also designed for all fighter planes to calculate the state of enemy plane. Despite the surrounding and encirclement control problems have been investigated in [10–12], the system models they considered were all integer-order, and there are no research about target-encirclement control of FOMASs so far. In this paper, the target-encirclement control of FOMASs is investigated. In contrast to [19–21], where the leader-following consensus problem of FOMASs were investigated, we consider the target-encirclement control for FOMASs with only one leader in this paper. Based on the theory of Mittag-Leffler stability instead of the conventional Lyapunov stability, a novel target-encirclement control law is proposed and some sufficient conditions are given for achieving target-encirclement control of FOMASs with a leader. At last, the effectiveness of the presented results were demonstrated by simulations.

In summary, the main contributions of this work are listed as follows:

- A novel target-encirclement control law is proposed for solving the target-encirclement control problem of FOMASs with a leader.
- Two state estimators are designed for each follower to estimate the state of the leader and the target respectively.
- Some sufficient conditions for achieving target-encirclement control of FOMASs are derived based on the theory of Mittag-Leffler stability.

The rest of this paper is organized as follows: In Section 2, some preliminaries about graph theory and fractional calculus are introduced, and then system models are given. Section 3 gives the main results of this paper, a novel target-encirclement control law is proposed, where the state estimators of the leader and the target are induced, and some sufficient conditions are given for achieving target-encirclement control of FOMASs. Numerical simulations are given in the next section to verify the theoretical analysis. Finally, we make a conclusion for this paper.

2. Preliminaries

2.1. Graph theory

The theory of algebra graph [25] is useful for the target-encirclement control problems. Consider a multi-agent system with N followers and one leader. The leader and every follower is seen as a node. The N followers are regarded as the vertices $\mathcal{V} = \{v_i, i = 1, 2, \ldots, N\}$, denote $\varepsilon \subset \mathcal{V} \times \mathcal{V}$ as the set of edges between follower nodes, given undirected graph $\mathcal{G} = \{\mathcal{V}, \varepsilon\}$ to describe the communication topology of the N followers. If $(v_i, v_j) \in \varepsilon$, then we say node v_i is the neighbor of node v_j, and we regard $N_j(t) = \{i|(v_i, v_j) \in \varepsilon, v_i \in \mathcal{V}\}$ as the set of labels of the neighbors of agent $j (j = 1, 2, \ldots, N)$. Let $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{N\times N}$ be the adjacency matrix of the graph $\mathcal{G}$, $a_{ij} = 1$ if and only if $(v_i, v_j) \in \varepsilon$, otherwise $a_{ij} = 0$, and $\mathcal{D} \in \mathcal{R}^{n\times n}$ be its degree matrix. The Laplacian of the graph $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, which is symmetric. A path connecting v_i and v_j in the graph $\mathcal{G}$ is a sequence of distinct vertices $v_{i_0}, v_{i_1}, \ldots, v_{i_m}$, where $v_{i_0} = v_{i_1}$, $v_{i_m} = v_{i_j}$ and $(v_{i_r}, v_{i_{r+1}}) \in \varepsilon$, $0 \le r \le m - 1$. The graph is said to be connected, if there exists at least one path between any two vertices v_i and v_j $(i \ne j)$. If we regard the leader as a vertex v_0, suppose d_i represents the communication weight between the leader and followers, and $d_i = 1$ if and only if the follower i can obtain the information from the leader, otherwise $d_i = 0$. Then the communication topology can be described by a graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\varepsilon})$, where $\bar{\mathcal{V}} = \mathcal{V} \cup \{v_0\}$, $\bar{\varepsilon} = \varepsilon \cup \{\epsilon_{d_i} : d_i = 1\}$, and ϵ_{d_i} be the directed edge from node v_i to v_0. Let $\mathcal{B} = diag\{d_1, d_2, \ldots, d_N\}$, $\mathcal{H} = \mathcal{L} + \mathcal{B}$, $\lambda_{min} = min\{\lambda(\mathcal{H})\}$, where $min\{\lambda(\mathcal{H})\}$ represent the minimum eigenvalue of matrix $\mathcal{H}$. $\mathbf{1}$ represents a column vector whose elements are all 1.