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# Critical spreading dynamics of parity conserving annihilating random walks with power-law branching

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## HIGHLIGHTS

- Branching and annihilating random walks with power-law branchings are considered.
- The critical spreading dynamics in d=1 is probed using finite-time scaling.
- The system presents a non-equilibrium phase transition with continuously varying exponents.
- Changes in the universality class are identified even within the effective diffusive regime.

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## ABSTRACT

We investigate the critical spreading of the parity conserving annihilating random walks model with Lévy-like branching. The random walks are considered to perform normal diffusion with probability $p$ on the sites of a one-dimensional lattice, annihilating in pairs by contact. With probability $1-p$, each particle can also produce two offspring which are placed at a distance $r$ from the original site following a power-law Lévy-like distribution $P(r) \propto 1/r^{\alpha}$. We perform numerical simulations starting from a single particle. A finite-time scaling analysis is employed to locate the critical diffusion probability $p_c$ below which a finite density of particles is developed in the long-time limit. Further, we estimate the spreading dynamical exponents related to the increase of the average number of particles at the critical point and its respective fluctuations. The critical exponents deviate from those of the counterpart model with short-range branching for small values of $\alpha$. The numerical data suggest that continuously varying spreading exponents sets up while the branching process still results in a diffusive-like spreading.



## 1. Introduction

Dynamical phase transitions present remarkable similarities with the usual equilibrium phase transitions observed in condensed matter systems [1,2]. They present a critical point which can be tuned by changing the system parameters and several quantities become singular at the transition with characteristic critical exponents that can be classified into a small group of universality classes. These exponents also satisfy a set of scaling relations. However, these systems usually follow dynamical stochastic rules that have no supporting Hamiltonian. As such, the fluctuation–dissipation theorem does not hold in general, and the critical exponents of the susceptibility and order-parameter fluctuations are distinct. Also, the

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absence of a natural partition function impairs the direct use of the Yang–Lee formalism, although some attempts along this direction have been proposed in the literature [3–7]. The most common dynamic phase transition is that between a statistically stationary active state and an absorbing vacuum state [1,2]. The simplest model presenting this kind of transition is named the contact process which presents competing death and branching processes. For small branching rates any initial configuration is driven to the absorbing vacuum state. Above a critical branching, the system evolves to a stationary state with a finite density of particles. This transition belongs to the universality class of directed percolation (DP), which is the most common class of absorbing phase transitions [1,2,8,9]. For the absorbing state transition taking place in a one-dimensional lattice, some relevant critical exponents are $\beta = 0.2767(4)$, $\gamma = 0.5438(13)$, $\nu_\perp = 1.0972(6)$, and $\nu_\parallel = 1.7334(10)$ [10–12], associated with the order parameter (density of particles), its fluctuations, the spatial and temporal correlation lengths, respectively. Other universality classes have been identified as, for example, for co-existing diffusive populations with a conserved field [13–19].

Branching and annihilating random walks (BARW) distributed in a lattice also present a dynamic transition between the vacuum and the active states [20,21]. In this model, particles can diffuse with probability $p$ or branch with the complementary probability $(1 - p)$. During the branching process, a number $n$ of offspring are generated. Particles annihilate each other when they try to occupy an already occupied site, either during the diffusion or branching processes. Whenever the number of offspring is odd, the absorbing state transiting belongs to the usual directed percolation universality class [20,22–24]. On the other hand, for an even number of offspring, the parity of the number of particles becomes a conserved quantity, which drives the system to a new universality class [20,25]. It is interesting to stress that the system always evolves to the vacuum state for the particular case of BARW in a one-dimensional lattice with $n = 2$ offspring placed on the two nearest neighbor sites [26]. An active state is only stable for $n \geq 4$ or for $n = 2$ with asymmetric branching [27]. In this case, the critical exponents deviate substantially from those of DP, with $\beta = 0.92(3)$, $\gamma = 0.00(5)$, $\nu_\perp = 1.84(6)$, and $\nu_\parallel = 3.25(10)$ [25].

Long-ranged processes are known to influence the critical behavior of equilibrium and non-equilibrium phase transitions [28,29]. One of the most common forms to introduce a long-range process in BARWs is to consider that the particles diffuse performing Lévy flights. In this case, instead of just jumping to neighboring sites, the particles can jump to a site placed at a distance $r$ following a power-law distribution $P(r) \propto 1/r^\alpha$. Usually, the power law exponent $\alpha$ is written as $\alpha = d + \sigma$, with $d$ being the lattice dimensionality. For the one-dimensional contact process with Lévy-like interactions, it has been shown that the critical behavior remains in the DP universality class for $\sigma > \sigma_c$, with $\sigma_c \simeq 2.07$, i.e., slightly above the regime of effectively super-diffusive dynamics [29,30]. Deviations from DP were reported for $1/2 < \sigma < \sigma_c$ with the exponents continuously varying with the Lévy exponent. These assume mean field-values for $\sigma < 1/2$. The influence of Lévy flights on the one-dimensional parity conserving BARW has also been investigated for the particular case of asymmetric $n = 2$ offspring [31]. The critical diffusion probability reaches $p_c = 1$ for $\sigma = 3/2$ $(\alpha = 5/2)$ with the system remaining in the active state for any finite branching probability for $\sigma < 3/2$ $(\alpha < 5/2)$. Continuously varying exponents takes place for $\sigma > 3/2$ up to a critical value of $\sigma$. Although the numerical data did not allow to firmly state this bound, field theoretical arguments suggests $\sigma_c \simeq 2.5$ for the parity-conserving DP class [29]. Above this bound, the exponents are equivalent to those of parity conserving BARWs with short-range interactions. The critical moment ratio of the order parameter was also shown to vary continuously with $\alpha$ [32]. However, the critical exponents reported were all associated with the long-time convergence to the absorbing state or to the ultimate stationary state. Studies of the critical spreading dynamics for the parity conserving BARW model are still missing.

In the present work, we advance in the characterization of the absorbing state phase transition exhibited by parity conserving BARWs by probing its critical spreading dynamics in the presence of long-ranged Lévy-like flights. We will focus on the particular case of asymmetric Lévy branching with $n = 2$ offspring. Starting with just one particle placed in a one-dimensional lattice, the system's trapping by the vacuum state will be avoid, thus allowing us to perform an accurate finite-time scaling analysis to compute the relevant spreading critical exponents associated with the average number of particles and its fluctuation. The dependence of the spreading critical exponents on the Lévy exponent will also be reported.

The manuscript is organized as follows: In Section 2 we define the model system considered, provide a detailed description of the simulations made, define the measured quantities and the finite-time scaling analysis employed to compute the spreading critical exponents. In Section 3 we provide our results for the critical spreading of the average number of particles and the time-evolution of its fluctuations, reporting the dependence of the critical parameters with the Lévy exponent. Finally, we summarize and draw our main conclusions and perspectives in Section 4.

## 2. BARW with parity conservation and Lévy-like branching

In the BARW model, particles are distributed on a given lattice an can diffuse or generate $n$ offspring. Double occupancy is forbidden and, consequently, particles annihilate in pairs when the dynamical processes drive two of them to the same site. In an elementary time step, a particle is chosen at random. It diffuses with probability $p$. Here, we will consider that the particle executes a short-ranged diffusion to one of its neighboring sites, chosen with equal probability. With probability $(1 - p)$ the particle remains on its position but generates $n$ offspring. Here we will consider the case of asymmetric $n = 2$ branching for which both offspring are placed at the same side of the particle (chosen at random). However, instead of placing the offspring on the nearest neighborhood, they are placed at distance $r$ and $r + 1$ from the original site. The distance $r$ is randomly chosen from a power-law Lévy-like distribution $P(r) \propto 1/r^\alpha$, with $\alpha$ being the exponent controlling the long-range character of the branching process.