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Discrete-time model for a motion of substance in a channel of a network with application to channels of human migration

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HIGHLIGHTS

- Discrete-time model for motion of substance in a network channel is discussed.
- Distributions of substance along the nodes of the channel are studied.
- Obtained distributions are generalizations of the Waring and Zipf distributions.
- Application of the model to finite channels of human migration is presented.
- Corresponding class of distributions for infinite channel is obtained.

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ABSTRACT

We discuss a discrete-time model for motion of substance in a channel of a network. For the case of stationary motion of the substance and for the case of time-independent values of the parameters of the model we obtain a new class of statistical distributions that describe the distribution of the substance along the nodes of the channel. The case of interaction between a kind of substance specific for a node of the network and another kind of substance that is leaked from the channel is studied in presence of possibility for conversion between the two substances. Several scenarios connected to the dynamics of the two kinds of substances are described. The studied models: (i) model of motion of substance through a channel of a network, and (ii) model of interaction between two kinds of substances in a network node connected to the channel, are discussed from the point of view of human migration dynamics and interaction between the population of migrants and the native population of a country.

1. Introduction

In the last decades models of flows in networks are much used in the study of different kinds of problems, e.g., transportation problems [1–7]. In the course of the years the research interest (that initially was focused on problems such as possible maximal flows in a network, minimal cost flow, or meeting fixed schedule with minimum number of individuals) expanded to the research areas of: just in time scheduling, shortest path finding, self-organizing network flows, facility layout and location, modeling and optimization of scalar flows in networks [8], optimal electronic route guidance in urban traffic networks [9], isoform identification of RNA [10], memory effects [11], epidemic spreading on lattices and trees [12–14], water flow networks [15], communication networks [16], supply chain engineering [17], etc. (see, e.g., [18–26]). We shall discuss

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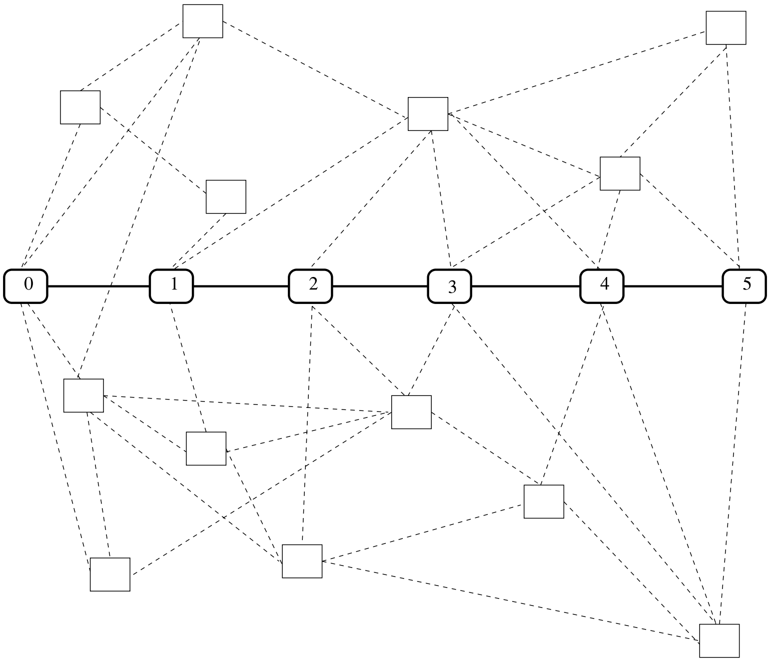

Fig. 1. A network and a channel. The channel consists of 6 nodes labeled from 0 to 5. The node 0 is the entry node of the channel (the substance enters the channel through this node). The nodes and the edges of the channel are marked by bold lines. The other nodes and edges of the network are represented by rectangles and dashed lines.

in this article a discrete-time model for the motion of a substance through a network channel in presence of possibility for "leakage" of substance. One possible application of the discussed model is for the flow of a substance through a channel with use of part of the substance in some industrial process in the nodes of the channel. However the model has more possible applications and we shall show this for the case of human migration flow. Human migration is an actual research topic that is very important for taking decisions about economic development of regions of a country [27–36]. Human migration is closely connected, e.g., to: (i) migration networks [37,38]; (ii) ideological struggles [39,40]; (iii) waves and statistical distributions in population systems [41–44]. We note that the probability and deterministic models of human migration are interesting also from the point of view of applied mathematics [45–57].

The text below is organized as follows. In Section 2 we discuss a discrete-time model for motion of substance in a channel containing finite number of nodes. A class of statistical distributions is obtained in Section 3. These distributions describe the distribution of the substance in the nodes of the channel for the case of stationary motion of substance through the channel. Particular cases of the distributions obtained in Section 3 and in Appendix are the distributions of Waring, Yule–Simon, and Zipf. In Section 4 we study the interaction between two kinds of substances in a node of the network. The substances are: (i) substance that is "native" for the node of the network, and (ii) substance that "leaks" from the node of the channel to corresponding node of the network. In Section 5 we apply the results from Section 4 to the case of interaction between population of migrants (the number of migrants may increase by inflow of migrants from the migration channel) and native population of a country. Several concluding remarks are summarized in Section 6. Appendix contains results for the class of statistical distributions that describe the distribution of substance along the nodes of the channel for the case of infinite length of the studied channel.

2. Mathematical formulation of the model

Let us consider a network consisting of nodes connected by edges. We assume existence of a channel in this network - Fig. 1. The structure of the channel and it relation to the network are as follows. Several of the nodes of the network together with the corresponding edges belong also to the channel. In Fig. 1 these nodes and edges are marked by bold lines. An exchange of substance between the channel and the network can happen in the nodes of the channel (denoted as "leakage" below). We shall assume that the processes in the network do not influence the flow of the substance in the channel. The "leakage" of the substance from a node of the channel however may influence the processes in the corresponding node of the network. We shall discuss such an influence in Section 4.

We assume further that the channel consists of a chain of $N+1$ nodes (labeled from 0 to N) connected by corresponding edges. Each edge connects two nodes and each node is connected to two edges except for the 0th node and Nth node that are connected by one edge. We assume that a substance can move through the channel. The substance enters the channel