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A stochastic model for making artificial rain using aerosols

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HIGHLIGHTS

- A deterministic model along with its stochastic version for artificial rain is proposed and analyzed.
- Multiplicative noise is introduced in the system to study effect of environmental fluctuations.
- Dispersion of variables depends on intensity of white noise.
- It is found that for small noise, system shows stationary distribution.
- Analytical results are supported by numerical simulation.

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ABSTRACT

In this paper, a nonlinear deterministic mathematical model along with its stochastic version for artificial rain is proposed and analyzed. We have considered three dynamical variables in the modeling process; namely (i) density of cloud droplets, (ii) density of raindrops, and (iii) concentration of mixture of conducive aerosols. It is assumed that the cloud droplets are continuously formed in the atmosphere at a constant rate but its conversion into raindrops does not take place in the same proportion. The artificially introduced aerosols increase the rate of formation of raindrops from cloud droplets. These aerosols are introduced in the regional atmosphere at a rate proportional to the density of cloud droplets. The proposed model is analyzed using stability theory of differential equations in deterministic as well as stochastic environment. Numerical simulation is performed to see the effect of important parameters on the process leading to rainfall.

1. Introduction

The shortage of water is a major cause of concern not only in India but all over the world. Several states of India, like Uttar Pradesh, Chhattisgarh, Maharashtra, Karnataka are much affected due to the shortage of water. The poor rainfall has created draught like situation in several parts of these states and the groundwater level has also fallen down. Simultaneously, this has also affected the other sources of water, like ponds, lakes, rivers, etc., as these sources are recharged during the rainfall. Most of the farmers in these states depend on rainfall to irrigate their agricultural farms. In the last few decades, due to inadequate rainfall, less production of grains, vegetables, fruits, etc., is observed and this has directly affected the income of farmers. It is reported that some farmers have committed suicide because of the reduced production from agricultural land. Thus, the inadequate rainfall leads to less grain production and affects not only the economic position of the farmers but also the country. To resolve the shortage of water, in the recent past some experimental and theoretical studies have been conducted to make artificial rain using aerosols in the local region.

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Precipitation includes a sequence of processes and it is complex to understand the actual mechanism followed in the process of precipitation. In the atmosphere, clouds consist of water droplets of very small size and for precipitation, these droplets need to grow through the processes of condensation and coalescence. About one million cloud droplets, which are usually of size 0.001 mm or lesser stick together to form a single raindrop. Generally, this process is slow and takes about one day to reach at full developed form. This is the reason that sometimes clouds are seen in the atmosphere but rain does not take place. By seeding clouds, the process of collision and coalescence can be accelerated to become droplets heavy enough to break buoyancy of cloud, resulting precipitation [1]. Some experiments have been performed to find suitable aerosols, which enhance the precipitation process and thus leading to artificial rain. For instance, Bruintjes et al. [2] have shown that the use of poly dispersed salt powder increases precipitation from cloud. In the same way Drofa et al. [3] have shown in their experiment that seeding with the salt powder in appropriate amount results the increment in the size and mass concentration of cloud droplets and the salt powder gives better result than hygroscopic flares to enhance rain. The same hypothesis is also supported by several other experiments mentioned in [4–7].

In the recent past, some efforts have been made to understand the role of aerosols to make artificial rain, using mathematical models. In this regard, Shukla et al. [8] have proposed a nonlinear mathematical model for making artificial rain by assuming that two kinds of aerosols are introduced in the atmosphere through aircrafts. It is also assumed that the first kind of aerosols are responsible for the formation of cloud droplets from water vapors; whereas the second kind of aerosols are responsible for the formation of raindrops from cloud droplets and both kinds of aerosols are introduced at a constant rate in the atmosphere. Further, this model has been generalized by considering the natural formation of cloud droplets from water vapors and raindrops from cloud droplets but for sufficient amount of rainfall, aerosols are introduced to enhance these processes [9]. Both the studies reveal that in the presence of water vapors, rainfall increases as the rate of introduction of both kinds of aerosols increases. More recently, Misra [10] has proposed a mathematical model by considering that a mixture of aerosols, conducive to the formation of cloud droplets from water vapors and raindrops from cloud droplets, is introduced in the local atmosphere at a constant rate from the ground. In this model, the removal of aerosols due to the impaction of rainfall is also considered apart from the natural formation of cloud droplets and raindrops. This model has been further extended by incorporating the size (small and large) of cloud droplets in the modeling process [11]. It is assumed that the small size cloud droplets are formed from water vapors and these small size cloud droplets stick together to form large size cloud droplets and finally the large size cloud droplets are converted to raindrops. In these studies, it is concluded that for the desired amount of rainfall, continuous introduction of aerosols plays an important role. Sundar and Sharma [12] have also proposed a three dimensional model to study the role of aerosols to increase the rainfall in the regions with shortage of rain. Some other mathematical models for the removal of gaseous pollutants from the environment, using rain have also been proposed and analyzed [13,14]. It is concluded that the concentration of pollutants in the atmosphere decreases as the intensity of rainfall increases, showing the importance of rainfall.

Here, it may be noted that all the above mentioned mathematical models for artificial rain are deterministic in nature and the random movements of water vapors, cloud droplets and aerosols are ignored. The incorporation of randomness in the weather forecasting models depict more realistic phenomenon and gives rise to stochastic differential equations [15,16]. However, the stochastic differential equations are also used in epidemiology to capture the movement of human population [17,18] and it is found that these models provide the results which are near to the real situation. To capture the random movements of gaseous pollutants in the environment, Huang et al. [19] have generalized the mathematical model proposed by Sundar et al. [20] for the removal of gaseous pollutants and the particulate matter from the environment. In this study, the asymptotic stochastic stability of the interior equilibrium is discussed.

Keeping the above facts in mind, in this paper, we propose and analyze a nonlinear mathematical model for making artificial rain in deterministic as well as stochastic setting. As the excess amount of aerosols (chemicals) used for making rainfall are hazardous to the Earth's environment [21] and so we assume that the rate of introduction of aerosols is proportional to the density of cloud droplets.

2. Mathematical model

In the present work, we propose a mathematical model for artificial rain using aerosols in the regional atmosphere. It is considered that the clouds are continuously formed in the atmosphere but rain does not take place. It is assumed that the aerosols are introduced in the atmosphere, which interact with cloud droplets and form raindrops and thus leading to rainfall. Let $C_d(t)$, $C_r(t)$ and $C_h(t)$ be the density of cloud droplets, density of raindrops and the cumulative concentration of externally introduced aerosols, respectively at any time t in the regional atmosphere. It is assumed that cloud droplets are continuously formed in the atmosphere at a constant rate Q_d and are depleted naturally proportional to their density due to natural conversion into raindrops, winds, etc. Since raindrops are formed by cloud droplets through collision, in this process water is conserved in the atmosphere [22,23], thus the growth rate in the density of raindrops is considered to be proportional to cloud droplets (i.e., rC_d). As pointed out above that the excess amount of chemical aerosols is hazardous to the Earth's environment, so the rate of introduction of aerosols in the atmosphere is assumed to be proportional to the density of cloud droplets (i.e., θC_d) and the aerosols depleted naturally from the atmospheric environment proportional to their concentration. When these aerosols interact with the cloud droplets, the density of cloud droplets and the concentration of aerosols both decrease and this rate of decrease is proportional to the density of cloud droplets and the concentration of aerosols (i.e., $\lambda_1 C_d C_h$), where λ_1 is the interaction rate coefficient. As the raindrops are formed due to this interaction and so