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Generalized information entropy analysis of financial time series

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HIGHLIGHTS

- We propose a Rényi permutation entropy (RPE) measure based on Shannon permutation entropy (PE) in stock markets.
- We extend Rényi permutation entropy (RPE) and weighted Rényi permutation entropy (WRPE) to multiscale analysis.
- MSWRPE retains most of PE's properties and shows more robustness to noise due to the existence of the parameter q.
- The properties of MSWRPE are distinct for $q > 1$ and $q < 1$.

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ABSTRACT

Generalized information entropy has been widely applied to analyzing complex systems. In this paper, we propose the weighted multiscale Rényi permutation entropy (MSWRPE) based on the weight assigned to each vector as a novel technique to consider the amplitude information. Rényi permutation entropy (RPE) has a parameter q for non-extensivity compared to Shannon permutation entropy (PE). Hence we speculate that RPE has a better sensitivity to patterns extracted from signals containing amplitude information and a better robustness to noise compared to PE. Firstly, we perform the multiscale Rényi permutation entropy (MSRPE) and MSWRPE methods on synthetic data. We find that MSWRPE suits better signals containing considerable amplitude information and is successful to consider the multiple time scales inherent in the financial systems. The finding is also verified in four different stock markets. Then, we make a comparison between MSWRPE and weighted multiscale permutation entropy (MSWPE) on different stock markets. The conclusion is that the MSWRPE method has a better characterization than MSWPE. For $q < 1$, different markets have the same law on MSWRPE, while HSI can be distinguished from the other markets for $q > 1$, which is more obvious when $m = 7$.

1. Introduction

Discriminating the presence of correlations in time series can be of great help when investigating the dynamics of systems from diverse scientific fields. Different measures of complexity were developed to compare time series and distinguish regular (e.g., periodic), chaotic, and random behaviors [1–5]. The main methods of complexity parameters are generalized information entropy, fractal dimensions [6–9], and Lyapunov exponents [10,11], etc. One of the most easy and practical method is the permutation entropy (PE) [12,13]. It is just the celebrated Shannon entropy measure evaluated using the successful recipe introduced by Bandt and Pompe (BP) to extract the probability distribution associated with an input signal [14,15].

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In the previous studies, many methods based on the PE have been suggested as a relative measure of complexity in different fields. Xia et al. applied the multiscale weighted permutation entropy (MSWPE) to traffic series and they found MSWPE can distinguish the differences between workday and weekend time series clearly [16]; Zhu et al. presented the delay permutation entropy (DPE) methodology to measure iEEG with different delay lag based on focal epileptogenic zone [17]. However, the main drawback of PE lies in the fact that no information besides the sequence structure is retained when extracting the ordinal patterns for each time series. The reasons why the shortcoming makes the analysis illogical can be listed as follows: firstly, the information containing in the amplitude in financial time series may be lost due to extracting the sequence structure easily; secondly, ordinal patterns where the amplitude differences between the time series are greater than others should not contribute similarly to the final PE value; finally, noise may produce small fluctuations in the time series and affect the ordinal patterns, so these ordinal patterns should not be weighted equally towards the final value of PE. As a result, we apply the weighted permutation entropy (WPE), which modifies the way PE handles the patterns extracted from a given signal by retaining amplitude information, to solving these problems. The weighted scheme makes it possible to discover the abrupt changes in the data and assigns more weight to the regular spiky patterns. That is to say, it is clearly able to distinguish between small fluctuations (may due to effect of noise) and large fluctuations of time series [18,19].

Considering that traditional entropy-based algorithms are single-scale and fail to account for the multiple time scales inherent in the financial systems, we need to extend PE and WPE to multiscale analysis in this paper. The multiscale analysis was proposed by Costa et al. to measure the complexity of biologic systems and was applied to analyzing other systems [20–22]. We noticed that definitions of all the above entropies are based on Shannon information theory, which belongs to a shortrange or extensive concept. In this article, we propose a novel generalized form of entropy: Rényi entropy. Rényi entropy has a parameter q for non-extensivity. If $q > 1$, the entropy is more sensitive to events that occur often, whereas if $0 < q < 1$ it is more sensitive to the events that occur seldom. In the limit $q \to 1$, it coincides with Shannon entropy. Rényi entropy algorithms have been widely used to detect dynamical changes in the physical systems especially the biomedical systems which are often characterized by either long-range interactions, long-term memories, or multifractality. The generalized entropy can provide additional information about the importance of specific events, such as outliers or rare events [23–27]. These features motivate us to explore whether this concept can be effectively used to analyze stock markets. Therefore, we extend Rényi permutation entropy (RPE) and weighted Rényi permutation entropy (WRPE) to multiscale analysis called multiscale RPE (MSRPE) and multiscale WRPE (MSWRPE) in this study.

In Section 2, we briefly introduce and calculate multiscale Rényi permutation entropy and weighted multiscale Rényi permutation entropy. Section 3 describes the database used in this paper. In Section 4, we employ the MSRPE and MSWRPE methods to analyze white Gaussian noise, ARFIMA series and actual stock markets, while we make a comparison between MSWRPE and MSWPE on different stock markets and obtain some interesting results. Finally, Section 5 presents the conclusion.

2. Methodologies

2.1. Rényi permutation and weighted Rényi permutation entropy

Consider the time series $\{x_i\}_{i=1}^{N}$ where N is the length of series and generate a vector $X_j^m = \{x_j, x_{j+1}, \ldots, x_{j+m-1}\}$ where m is the embedding dimension. Each sequence X_j^m is sorted in ascending order with permutation pattern π_i^m and there will be $m!$ possible permutations. RPE of order m is then defined as:

$$R_q(m) = \frac{1}{1-q}\log(\sum_{i:\pi_i^m \epsilon \Pi} p(\pi_i^m)^q) \tag{1}$$

where Π denotes the $\{\pi_i^m\}_{i=1}^{m!}$ and $p(\pi_i^m)$ is calculated as:

$$p(\pi_i^m) = \frac{\{j | j = 1, \ldots, N-m+1; X_j^m \text{ has type } \pi_i^m\}}{N-m+1} \tag{2}$$

where the order q ($q \geq 0$ and $q \neq 1$) is a bias parameter: $q < 1$ privileges rare events, while $q > 1$ privileges salient events. The Shannon entropy S[P] is recovered in the limit as $q \to 1$. RPE offers a more flexible tool, allowing for a better characterization of the process under study than just the Shannon permutation entropy counterpart.

The procedure of calculating the WRPE is briefly described here: Each vector X_j^m is weighted with the weight value w_j, instead of being weighted uniformly. The weighted relative frequencies are calculated as follows:

$$p_w(\pi_i^m) = \frac{\sum_{j:X_j^m \text{ has type } \pi_i^m} w_j}{\sum_{j \leq N-m+1} w_j}. \tag{3}$$

Note that $\sum_i p_w(\pi_i^m) = 1$. The weight value w_j is the variance of each vector X_j^m as

$$w_j = \frac{1}{m}\sum_{k=1}^{m}[x_{j+k-1} - \bar{X}_j^m]^2 \tag{4}$$