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An appetizer to modern developments on the Kardar–Parisi–Zhang universality class

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HIGHLIGHTS

- A review on recent developments on KPZ, intended for non-specialists, is given.
- Physical implications of theoretical results are stressed.
- Connections to directed polymer, a quantum many-body system, etc. are explained.
- Experimental study using liquid-crystal turbulence is also explained.

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ABSTRACT

The Kardar–Parisi–Zhang (KPZ) universality class describes a broad range of non-equilibrium fluctuations, including those of growing interfaces, directed polymers and particle transport, to name but a few. Since the year 2000, our understanding of the one-dimensional KPZ class has been completely renewed by mathematical physics approaches based on exact solutions. Mathematical physics has played a central role since then, leading to a myriad of new developments, but their implications are clearly not limited to mathematics — as a matter of fact, it can also be studied experimentally. The aim of these lecture notes is to provide an introduction to the field that is accessible to non-specialists, reviewing basic properties of the KPZ class and highlighting main physical outcomes of mathematical developments since the year 2000. It is written in a brief and self-contained manner, with emphasis put on physical intuitions and implications, while only a small (and mostly not the latest) fraction of mathematical developments could be covered. Liquid-crystal experiments by the author and coworkers are also reviewed.

1. Introduction

Physics of critical phenomena, as represented by those of the Ising model, is one of the monuments of statistical physics. From the author's possibly biased view, it was founded cooperatively by theoretical studies [1], both rigorous and non-rigorous, as well as by experiments [2]. More precisely, when Andrews discovered the liquid–vapor critical point in the 19th century [3], what he observed was the Ising critical behavior from the modern viewpoint, though the Ising model was invented much later. In the mid 20th century, the two-dimensional Ising model was solved exactly by Onsager, and subsequently by Kaufman and Nambu [4–7]. This exact solution clearly indicated the existence of nontrivial critical behavior that is different from the prediction of the mean field theory. Moreover, it turned out that critical phenomena exhibit certain extent of universality, as suggested by experimental observations of liquid–vapor systems, binary liquid mixtures and magnets [2]. This universality was clearly accounted for by Wilson's renormalization group [8] on the basis of continuum

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equations such as the ϕ^4 model, which allowed us to classify critical phenomena in terms of universality classes. This line of research marked further milestones, such as the foundation of conformal field theory [9], and it continues doing so, as exemplified by recent developments of conformal bootstrap theory [10] tackling the three-dimensional Ising problem.

All these developments concern critical phenomena at thermal equilibrium. Then, does there exist such a beautiful physics for systems driven out of equilibrium? No one knows the answer yet, but developments that might resemble the dawn of the equilibrium counterpart can be found in recent studies on the Kardar–Parisi–Zhang (KPZ) universality class [11–17]. The KPZ class, best known as the simplest generic class for fluctuations of growing interfaces [11], is also related to a wide variety of non-equilibrium fluctuations, including directed polymers, stirred fluids, and particle transport to name but a few. Furthermore, for the one-dimensional (1D) case, mathematical studies have unveiled nontrivial connections to random matrix theory, combinatorial problems, and integrable systems. This has driven intense activities to investigate exact fluctuation properties of the 1D KPZ class and its mathematical structure behind, providing a great number of exact results [14–18] which also have experimental relevance [19–21].

The aim of these lecture notes is to tempt non-specialists into this rapidly evolving field around the KPZ class. It is *not* a review of recent mathematical approaches nor a technical guide to solve the problems, for which the readers are referred to more appropriate reviews [14,15,17,18] and references therein. Instead, it is intended to provide useful information for non-specialist physicists to join the game, in a brief but self-contained manner: how the KPZ class is linked to various problems, what the main outcomes and implications for physicists are, and what kind of intuitions we can use, from the limited view of an experimentalist admirer and user of those mathematical developments.

These lecture notes are organized as follows. Section 2 describes what kind of interfaces we deal with, on the basis of some experimental observations. Section 3 introduces continuum equations and their associated universality classes, including the KPZ equation and the KPZ class, and review their basic properties. Properties of the KPZ equation are further described in Section 4. Sections 5 and 6 illustrate some exact results for the 1D KPZ class, focusing on the distribution of interface fluctuations. Section 7 reviews experimental observations of growing interfaces in turbulent liquid crystal, to be compared with the exact results described in the preceding sections. In Section 8 we briefly discuss the situation in higher dimensions. Section 9 provides brief concluding remarks.

2. Examples of quiescent and growing interfaces

The surface of water at rest is a symbol of something flat and smooth. Indeed, according to the capillary wave theory [22], which accounts for thermal excitations of such a free interface between two fluid phases, the amplitude of fluctuations under gravity is in the order of $\sqrt{k_{\mathrm{B}}T/\gamma}$ with the Boltzmann constant k_{B}, temperature T, and interfacial tension γ. For molecular fluids, it is usually shorter than 1 nm [22,23]; therefore, the interface is sufficiently smooth when observed at macroscopic length scales. But what happens if one of the phases is more stable than the other, hence taking over the region of the metastable state? What happens if one of the phases is solid and molecules are deposited on it one after another, such as in thin film growth? What happens if there is an aggregate of particles that can replicate themselves, such as living cells? Or, conversely, what happens if particles are being removed from the surface of the aggregate? In all such cases the interface (or the edge of the aggregate) will move in either direction, typically with fluctuations growing in time.

Let us see some examples. Fig. 1(a) shows snapshots of colonies of human cancer cells (HeLa), cultured in Petri dishes [24]. Those cells were initially cultured with an obstacle attached on the Petri dish bottom, which set the initially straight border of colonies. When the obstacle was removed, the colony starts to expand because of cell division and motility. As a result, the interface moves in the normal direction, but fluctuations also start to develop, because of the stochastic nature of cell behavior. Then the snapshots in Fig. 1(a) suggest that those interface fluctuations gain larger and larger structure as time elapses.

Fig. 1(b) shows another example, taken from an experiment of "coffee ring effect" [27] – a familiar phenomenon observed when a droplet of coffee is dried on a solid surface, which leaves a ring of coffee particles on the surface. The formation of the ring is due to the deposition of particles onto the droplet edge, carried by fluid capillary flow. In Fig. 1(b), polystyrene beads were used as particles, and the deposition was monitored by a microscope (see also Supplementary Movie 3 of [27]). One can again see that, as the front of the deposited particles moves inward, its fluctuations develop, because of random deposition of particles. Here we can also see that those particles interact; sometimes they collide and stick to each other, and in fact they can interact through deformation of the air–liquid surface.

In the literature, such kinetic roughening of interfaces has been reported in a wide variety of growth processes [12,13,29], classical examples being bacteria colonies on agar, slow combustion of paper, growth of solid thin film, etc. Kinetic roughening seems to be a common feature of interfaces that grow through short-ranged interactions under the influence of noise, whether it is inherent to dynamics (such as cell division) or results from heterogeneity of the system (such as fibrous structure of paper).[1] Those interfaces are not merely similar in the qualitative sense, but in many cases they have a striking quantitative property, namely, scale invariance. The example of cancer cell colony growth in Fig. 1(a) beautifully shows that, as time t is increased, both the amplitude of interface fluctuations δh and the length of correlation in the spanwise direction

[1] In the presence of non-local interactions, kinetics and resulting morphology of interfaces can be very different. For example, if particles can diffuse freely in the surrounding environment before they adhere to the interface, the growth is controlled by the density field of diffusing particles, which is affected by the global structure of the interface. Fractals tend to appear in such a case; see [12].

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