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Transition to turbulence in shear flows

Bruno Eckhardt

Fachbereich Physik, Philipps-Universität Marburg, 35032 Marburg, Germany

HIGHLIGHTS

- Summary of the steps towards turbulence in pipe flow.
- Describes non-normal amplification and sensitivity to perturbations.
- Describes secondary bifurcations and the origins of transient turbulence.
- Describes relation to spatiotemporal chaos and directed percolation.
- Outlines extensions to other shear flows.

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ABSTRACT

Pipe flow and many other shear flows show a transition to turbulence at flow rates for which the laminar profile is stable against infinitesimal perturbations. In this brief review the recent progress in the understanding of this transition will be summarized, with a focus on the linear and nonlinear states that drive the transitions, the extended and localized patterns that appear, and on the spatio-temporal dynamics and their relation to directed percolation.

1. Introduction

The ways in which flows become turbulent can broadly be divided into two groups. One group contains all flows where the laminar profile shows a linear instability as the flow rate increases. A subsequent cascade of instabilities results in ever more complicated dynamics, which in spirit, though not in detail, reflects Landau's descriptions of the transition to turbulence [1]. Fluids sheared between independently rotating concentric cylinders (Taylor–Couette flow) or heated from below (Rayleigh–Benard flow) follow this route and the linear and nonlinear properties of the patterns that form have been explored in considerable detail [2–6].

The second group has pressure driven flow in a pipe as a paradigmatic example [7,8], but also includes plane Couette flow or boundary layers and several other cases shown in Fig. 1. They all share the feature that turbulence appears for parameters where the laminar profile is still stable. Their transition typically requires finite amplitude perturbations, it shows a sensitive dependence on initial conditions, and there are no simple patterns above the onset [9]. Moreover, the flows show a remarkable variety of spatio-temporal dynamics near onset [10–14].

In the last two decades, various elements for an explanation of the transition in these flows have been identified and explored, so that we now have a framework in which pipe flow and other flows can be approached. The following sections provide a brief survey of key concepts such as exact coherent states (ECS), secondary bifurcations and the formation of turbulent transients and spatio-temporal patterns, and the connection to phase transitions of the directed percolation type. Various other aspects of the transition are discussed in [15–24].

E-mail address: bruno.eckhardt@physik.uni-marburg.de.

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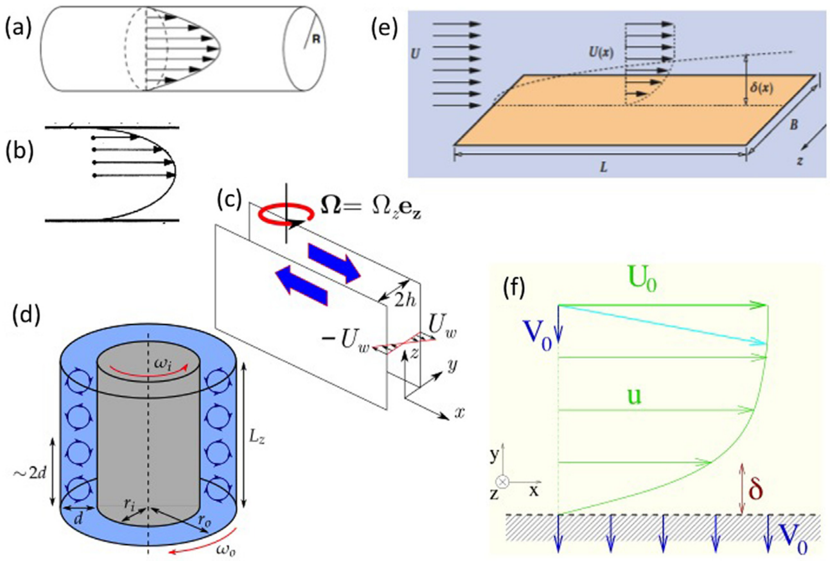

Fig. 1. (Colour online) Examples of flows where the transition to turbulence is induced by sufficiently strong perturbations and is not connected with a linear instability of the laminar profile. The left column shows internal flows bounded by walls on two sides, from top to bottom: (a) pressure driven flow in a cylindrical pipe of circular cross section (Hagen–Poiseuille flow); (b) the related pressure driven flow between parallel plates (plane Poiseuille flow); (c) shear flow between parallel walls moving relative to each other on a rotating table (rotating plane Couette flow); (d) flow between two independently rotating concentric cylinders (Taylor–Couette flow). The right column shows external flows bounded by a wall on one side only: (e) flow over a flat plate where the boundary layer increases with distance from the edge (Blasius) and (f) the suction boundary layer where a cross-flow maintains a parallel boundary layer of constant thickness. Of the flows shown here, plane Poiseuille flow, Taylor–Couette, and both boundary layer flows also have linear instabilities that become relevant in certain parameter ranges. The focus here is on transitions in parameter ranges without linear instabilities. Coordinates are usually chosen such that x points in the flow direction, y in the direction with the shear and z in the spanwise neutral direction.

In the following sections, we describe the non-normal amplification in the linearized dynamics (Section 2), finite amplitude subcritical bifurcations that give rise to exact coherent structures around which turbulence can form (Section 3), the presence of localized exact coherent structures (Section 4), the long-time dynamics of localized structures and their relation to directed percolation (Section 5). A few concluding remarks and an outlook on open issues are given in Section 6.

2. Linear approaches

The linearization of the Navier–Stokes equation around a background shear $\mathbf{U}_0$ reads

$$\partial_t \mathbf{u} + (\mathbf{U}_0 \cdot \nabla)\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{U}_0 + \nabla p = \nu \Delta \mathbf{u}\,. \tag{1}$$

Asymptotic stability requires that all perturbations decay eventually. This does not imply that perturbations decay monotonically: they can transiently be amplified beyond their initial amplitude before they disappear eventually. Typically, the structures that are most efficient in extracting energy from the background shear are downstream vortices which then drive the formation of modulations in the downstream velocity, so-called streaks. This interaction provides the building blocks for the nonlinear states as indicated in Fig. 2.

The significance of vortices and streaks can be studied in a simple model [25]. Consider a shear flow with x pointing in the direction of flow, y in the normal direction with the shear and z in the spanwise direction. Then the background velocity field with a linear shear is given by $\mathbf{U}_0 = Sy\mathbf{e}_x$. Adding as a perturbation the superposition of a vortex ω and a streak σ,

$$\mathbf{u} = \omega(t)\begin{pmatrix} 0 \\ \beta \sin\alpha y \sin\beta z \\ \alpha \cos\alpha y \cos\beta z \end{pmatrix} + \sigma(t)\begin{pmatrix} -\beta \sin\alpha y \sin\beta z \\ 0 \\ 0 \end{pmatrix}, \tag{2}$$

the resulting equations for the perturbation become

$$\partial_t \begin{pmatrix} \sigma \\ \omega \end{pmatrix} = \begin{pmatrix} -\Lambda & S \\ 0 & -\Lambda \end{pmatrix}\begin{pmatrix} \sigma \\ \omega \end{pmatrix}, \tag{3}$$

with the damping constant $\Lambda = \nu\left(\alpha^2 + \beta^2\right)$. For the initial condition $(\sigma, \omega)(0) = (0, 1)$ of a vortex but no streak, the time evolution becomes

$$\begin{pmatrix} \sigma(t) \\ \omega(t) \end{pmatrix} = \begin{pmatrix} Ste^{-\Lambda t} \\ e^{-\Lambda t} \end{pmatrix}. \tag{4}$$

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