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On the applicability of the Lead/Lag Ratio in causality assessment

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HIGHLIGHTS

- We present an in-depth validation of the LLR metric.
- LLR presents advantages like its conceptual simplicity and low computation cost.
- It also presents serious limitations, as the lack of a statistical significance test.

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ABSTRACT

Within the large set of metrics that have been proposed to assess the presence of a causality relationship between time series, the Lead/Lag Ratio (LLR) has recently attracted increasing attention, especially in the econophysics community. It is based on the analysis of the asymmetry between the correlations obtained when one time series is shifted back or ahead in time with respect to the second one; or, in other words, on the assessment of whether one time series is leading the other. In spite of its popularity, undoubtedly due to its conceptual and computational simplicity, no empirical validation of the LLR has hitherto been proposed. We here take a first step towards this validation, by presenting results corresponding to a large set of analyses on synthetic time series, and comparing them with those yielded by the celebrated Granger Causality metric. We show how, in spite of behaving similarly to the Granger Causality in some specific situations, the LLR presents important drawbacks, among which its unreliableness and the absence of a statistical significance test stand out, that discourage its use in real-world analyses.

1. Introduction

The analysis of time series, and more specifically of the relationships appearing between pairs of them, is a major topic of research in many scientific and technical fields. If one may initially rely on simple linear correlations, real-world problems usually require the identification of specific types of relationships, *e.g.* non-linear ones; this has resulted in the development of multiple, discipline-specific metrics. A special attention has been devoted to the detection of causality relationships, *i.e.* situations in which one time series drives (and hence controls the future of) a second time series [1]. Causality goes well beyond simple correlation: while the latter may indicate the presence of a direct relationship between two time series, a high correlation can also be the result of confounding effects, whose presence should be discarded to ensure a meaningful analysis [2].

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If a large number of metrics have hitherto been proposed to detect the non-trivial causality relationships between sets of coupled time series [3], a new metric bridging correlation and causality has recently been proposed: the *Lead/Lag Ratio* (LLR) [4]. This metric is based on the analysis of the correlation observed between two time series, considering different lag values; and on assessing whether the past of the driving time series is more correlated with the future of the driven one than the opposite situation. LLR's attractiveness thus resides in the capacity of using correlation (a very simple and well understood concept) to assess causality — something usually more obscure and ill-defined. Since its definition, LLR has been used to assess causality in different financial markets — see for instance [5–8]; additionally, it has recently been extended to frequency domain problems [9].

In spite of LLR's increasing popularity, some questions remain open regarding its validity and usability, as for instance: under which conditions can this metric detect causality? Is the metric influenced by the presence of confounding effects, like spurious correlations? Can an assessment of the statistical significance of results be developed? How does the LLR compare with other classical causality metrics, *e.g.* the Granger Causality (GC) [10,11]?

In this contribution we tackle these questions by performing a large set of analyses using synthetic time series, and by comparing the obtained results with those yielded by the Granger Causality metric. Among the large number of available metrics, this latter has been chosen as benchmark due to its widespread use in econometrics [12] and beyond [13,14]. Aspects taken into account include the properties of the time series under analysis, as their length and stationarity, and of the coupling between them, *e.g.* linearity, memory length, *etc*. We show that, while sometimes equivalent to GC and surely of greater simplicity (both conceptually and computationally), the LLR presents some important drawbacks, as its unreliableness and the absence of a suitable test to assess the statistical significance of results.

2. LLR definition

The Lead/Lag Ratio is built on top of the same principle buttressing the well-known Granger Causality metric: given two time series X and Y, the former is *causing* the latter only if knowledge about the past of X helps predicting the future evolution of Y. While Granger Causality assesses this by constructing autoregressive (AR) models on top of the time series, in the case of LLR, these AR models are substituted by time-shifted correlations.

More specifically, the LLR can be explained as follows. If the time series X is causing Y, one can expect Y_i (i designating one point in time) to be partly defined by a past element of X, *i.e.* by $X_{i-\delta}$. If the relationship between X and Y is linear, one would then expect to find a correlation between Y and X when the latter is shifted *ahead* in time by δ steps. On the other hand, the correlation between Y and X, when the latter is shifted *back* in time, should be negligible (or at least lower in magnitude), as future elements of X do not drive the past dynamics of Y.

Mathematically this can be formalised as follows:

$$LLR_{X\to Y} = \frac{\sum_{h>0} \rho^2(h)}{\sum_{h<0} \rho^2(h)}, \tag{1}$$

where $\rho^2(h)$ is the square of the correlation coefficient between X and Y, when the latter is shifted h steps forward in time (respectively backward for negative values of h). As the correlation for positive shifts of X should be lower than that for negative shifts, Ref. [4] claims that a causality relation between X and Y can be accepted when:

$$LLR_{X\to Y} < 1. \tag{2}$$

The Lead/Lag Ratio presents several advantages that make it a potentially interesting instrument for time series analysis in real-world applications. First of all, it is computationally inexpensive, as it reduces to an integration over the values yielded by the cross-correlation between the two time series; this also guarantees that the LLR can be implemented in any data analysis programming language with very few lines of code. Additionally, the LLR reduces the concept of causality to that of correlation, thus simplifying its intuitive understanding.

Nevertheless, the Lead/Lag Ratio also presents some important drawbacks, that are analysed in the next section.

3. LLR analysis

3.1. Statistical significance and normalisation

One of the first problems encountered when applying the LLR metric to real-world problems is that, as defined in Eq. (1), it does not provide a way for assessing the statistical significance of results. The output is not a p-value, as in the case of the Granger Causality; and the LLR is further defined in an asymmetrical way, with the presence and absence of a causality relationship respectively represented by the intervals $[0, 1)$ and $[1, \infty)$. While not essential (as will be further discussed below and in the conclusions), the availability of a p-value helps obtaining sound results when only one single realisation of the system is available. We thus here tackle this issue, by analysing the feasibility of defining one for the LLR.

As a first step, we tackle the problem by redefining the LLR metric as:

$$LLR^*_{X\to Y} = \log_{10} LLR_{X\to Y} = \log_{10} \frac{\sum_{h>0} \rho^2(h)}{\sum_{h<0} \rho^2(h)}. \tag{3}$$