Physica A 505 (2018) 105–113

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

A car-following model accounting for probability distribution

Hui Ou [a], Tie-Qiao Tang [b,*], Jian Zhang [b], Jie-Ming Zhou [a]

[a] Key Laboratory of High Performance Computing and Stochastic Information Processing (HPCSIP) (Ministry of Education of China), College of Mathematics and Computer Science, Hunan Normal University, Changsha, Hunan 410081, China

[b] School of Transportation Science and Engineering, Beijing Key Laboratory for Cooperative Vehicle Infrastructure Systems and Safety Control, Beihang University, Beijing 100191, China

HIGHLIGHTS

- A car-following model with probability distribution is proposed.
- The effects of the probability distribution on uniform flow are studied.
- The effects of the probability distribution on traffic waves are studied.
- The effects of the probability distribution on small perturbation are studied

ARTICLE INFO

Article history:
Received 26 December 2017
Received in revised form 17 February 2018
Available online 29 March 2018

Keywords:
Car-following model
Perceived error
Probability distribution

ABSTRACT

Various stochastic factors (e.g., the driver's individual properties) widely exist in the real traffic system, but the existing studies cannot completely describe the impacts of various stochastic factors on traffic flow (especially the driving behavior). In this paper, we introduce the driver's three perceived errors into the car-following model, and construct a car-following model with the probability distributions of the three perceived errors to explore the effects of the three perceived errors on the driving behavior under three typical situations (i.e., uniform flow, shock and rarefaction waves, and a small perturbation). The numerical results show that the three perceived errors have significant impacts on the evolution of traffic flow (including the headway distribution), i.e., the distribution of density does not prominently change under the three traffic states. In addition, the impacts are directly related to the initial condition. The results can help drivers reasonably adjust their driving behaviors based on their current traffic state (especially when some stochastic factors exist).

1. Introduction

Nowadays, researchers have proposed various traffic flow models to explore the serious traffic problems (e.g., congestion, accident, pollution, etc.) from different perspective [1,2]. Roughly speaking, the models can be classified into macroscopic ones [3–24] and microscopic ones [25–47]. The macroscopic ones use macroscopic variables (e.g., density and speed) to study the distribution, evolution and propagation of traffic flow, which are formulated by some partial differential equations (PDEs) [3–24]. The microscopic models use some microscopic variables (e.g., the variables of each vehicle/driver) to the micro driving behaviors and the corresponding complex traffic phenomena, which are formulated by a series of ordinary differential equations (ODEs) [25–47].

* Corresponding author.
E-mail address: tieqiaotang@buaa.edu.cn (T.Q. Tang).

https://doi.org/10.1016/j.physa.2018.03.072
0378-4371/

However, the above models do not consider the stochastic factors, so they cannot be used to directly study the complex traffic phenomena that are caused by the stochastic factors. In fact, many stochastic factors exist in a traffic system. For example, each driver may randomly adjust his/her driving behavior (e.g., speed) due to some specific conditions. Hence, the stochastic factors should be considered in a traffic flow model (especially in a driving behavior model). To explore this topic, researchers proposed some traffic flow models accounting for some stochastic factors (e.g., the driver's attribution), and used numerical tests to testify that the proposed models can reproduce the complex phenomena caused by some stochastic factors (including the effects of the stochastic factors on the micro driving behaviors) [48–50]. However, the models have two limitations, i.e., one is that most researchers did not use random variables to depict the stochastic factors, and the other is that most researchers used some random variables with uniform distribution. In addition, little effort has been made to study the impacts of the driver's perceived errors of headway, speed difference and acceleration on driving behavior (especially in the case that the three errors follow normal distribution). In this paper, we develop an extended car-following model with the driver's perceived errors of headway, speed difference and acceleration, where each error can simply be assumed as a random variable with normal distribution. The organization of this paper is set as follows: the car-following model with the three perceived errors is proposed in Section 2; some numerical tests are carried out to study the influences of the three errors on the driving behaviors under three typical situations in Section 3; and conclusions are summarized in Section 4.

2. Model

Many car-following models have been proposed to study the driving behaviors under many complex traffic situations, such as OV model [25], GF model [26], FVD model [27], and so on. These car-following models have different formulations, but they can be simplified as follows:

$$a_n = \varphi\left(\Delta x_n, v_n, \Delta v_n, \ldots\right), \tag{1}$$

where $a_n, v_n, \Delta x_n\,(n > 1) = x_{n-1} - x_n, \Delta v_n\,(n > 1) = v_{n-1} - v_n, x_n$ are the nth vehicle's acceleration, speed, headway, speed difference and position, respectively; φ is the nth vehicle's stimulus function that is determined by v_n, Δx_n, Δv_n and other related factors. We can deduce the FVD model [27] if Eq. (1) is defined as follows:

$$a_n = \kappa\left(V\left(\Delta x_n\right) - v_n\right) + \lambda \Delta v_n, \tag{2}$$

where κ, λ are two reaction coefficients; V is the nth vehicle's optimal speed determined by Δx_n.

The acceleration, headway, speed and speed difference in Eqs. (1) and (2) do not include each driver's corresponding perceived error, which shows that some stochastic factors are not considered in the above models. In fact, stochastic factors widely exist in real traffic system. For example, when different drivers evaluate headway and speed difference, they may have different values, which indicate that the differences should explicitly be considered in a driving behavior model. For convenience, $\Delta\overline{x}$, $\Delta\overline{v}$ are set as the perceived headway and speed difference, which can be defined as follows:

$$\Delta\overline{x} = \Delta x + \xi, \tag{3}$$

$$\Delta\overline{v} = \Delta v + \eta, \tag{4}$$

where Δx, ξ are respectively the real headway and perceived error; Δv, η are respectively the real speed difference and perceived errors. Here, ξ, η are stochastic variables.

Based on the above discussions, we should incorporate the perceived headway and speed difference into the car-following model. Thus, we can obtain a car-following model accounting for the driver's perceived errors of headway, speed difference and acceleration, i.e.,

$$a_n = \kappa\left(V\left(\Delta x_n + \xi_n\right) - v_n\right) + \lambda\left(\Delta v_n + \eta_n\right) + \varepsilon_n, \tag{5}$$

where ξ_n, η_n, ε_n are the perceived errors of headway, speed difference and acceleration of the nth driver, respectively (here, ξ_n, η_n, ε_n are stochastic variables); κ, λ are two parameters that can be defined as follows [27]:

$$\kappa = 0.41, \lambda = \begin{cases} 0, & \text{if } \Delta x_n + \xi_n > 100\text{ m} \\ 0.5, & \text{otherwise.} \end{cases} \tag{6}$$

As for the above three stochastic factors, we should here give a note: Eqs. (1) and (2) show that acceleration, speed and speed difference are the three main variables that determine the each vehicle's motion, so we here only consider the stochastic features of the three variables. In fact, we can introduce other stochastic variables in the car-following model and obtain some similar results.

Next, we should discuss the features of the three stochastic variables in Eq. (5). Based on the above discussions, it is very easy to prove that the expected values of the three stochastic variables are 0. In fact, drivers are randomly distributed on each road, but ξ_n, η_n, ε_n may have many different distributions. For simplicity, we here assume that ξ_n, η_n, ε_n follow normal distribution, where their standard deviations are respectively σ_{ξ_n}, σ_{η_n}, σ_{ε_n}. Based on the features of normal distribution and the physical meanings of ξ_n, η_n, ε_n, we define σ_{ξ_n}, σ_{η_n}, σ_{ε_n} as follows:

$$\sigma_{\xi_n} = g_{\xi_n}\left(\Delta x_n\right), \tag{7}$$