Physica A 503 (2018) 322–330

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Analysis of feedback control scheme on discrete car-following system

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HIGHLIGHTS

- Newell's continuous car-following model was discretized into a difference equation.
- Sampling interval and feedback coefficient influences play important roles in the stability of system.
- Local feedback control scheme was designed in a discrete car-following system.

ARTICLE INFO

Article history:
Received 16 November 2017
Received in revised form 11 February 2018
Available online 7 March 2018

Keywords:
Discrete car-following model
Feedback control
Routh criterion

ABSTRACT

We discretized the Newell's car-following model into a difference equation and the impulse transfer function is derived. The stability condition is given and proven as a Lemma based on bilinear transformation and Routh criteria. Feedback control scheme was applied to improve the performance of car-following system. A theorem was proposed to judge the stability condition in designing the feedback coefficient for controlled car-following system. Numerical simulations were conducted to verify the validity of the discrete car-following model and its control scheme. The simulation results show that the sampling time and feedback control coefficient have effects on the stability of car-following system.

1. Introduction

Car-following model as one of microscopic traffic flow models has been developed for many decades and was widely used to study the moving behavior of individual car in a system. Pipes [1] firstly proposed the classical car-following model to formulate a stream of cars' behavior in 1953. After that, so many improved car-following models were proposed to investigate the dynamics of traffic flow. Newell [2] proposed a first-order derivative equation with a delay time and optimal velocity function to describe the car-following behavior. Bando et al. [3] improved the Pipes' car-following model by substituting the front car's velocity with the optimal velocity function and overcame the shortcoming of the classical models. Holland [4] developed a general criterion to determine whether a catastrophic event will occur. The general criterion was in accordance with the stability condition of the car-following models. Nagatani [5] derived a difference equation to describe the car-following behavior, which was easily used to do computer simulation. In addition, Nagatani [6] and Sawada [7] improved the optimal velocity model by considering the next-nearest-neighbor interaction. The stabilization of the traffic flow was greatly enhanced and the solution of the mKdV equation was obtained. Based on Bando's model, Lenz et al. [8] considered multi-anticipative interaction and the stability region increases. Jiang et al. [9] proposed a full velocity difference

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https://doi.org/10.1016/j.physa.2018.02.192
0378-4371/

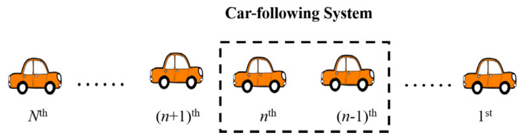

Fig. 1. Illustration of car-following system with N vehicles.

model for car-following theory. The model was more effective in describing the traffic flow testified by the analytical and numerical analysis. Konishi et al. [10,11] designed decentralized feedback control schemes to govern the car-following system. Modern control theory and frequency domain viewpoint were used to discuss the feasibility of control system. It is verified that the results are valid. Zhang et al. [12] proposed a new car-following model to describe the multiphase traffic. The so-called capacity drop and traffic hysteresis loop was reproduced. Jin et al. [13] proposed a delayed-feedback control of both displacement and velocity differences. It was effective in suppressing traffic jam. Li et al. [14] systematically discussed the local stability and asymptotic stability of the car-following model. Moreover, the corresponding Lyapunov stability is also proposed based on control viewpoint. Ge et al. [15] applied the delay-feedback control scheme to improve the traffic flow behavior. Tang et al. [16] proposed a new extended version car-following model by considering the communication of inter-vehicle and its liability. Ge et al. [17] and Redhu et al. [18] respectively applied the delay-feedback control scheme to improve the traffic flow performance. These methods were proven to be effective in suppressing the traffic jam. Yu and Shi [19,20] proposed two new connected cruise control strategies considering multiple preceding cars' velocity changes with memory and vehicular gap fluctuation respectively to improve roadway traffic mobility, to enhance traffic safety and to reduce fuel consumptions and exhaust emission. They verify that these models are valid through the numerical simulation. Zhu et al. [21] proposed a compound-compensation control strategy for car-following model based on cascade compensation. The compensation coefficients play important roles in stabilizing the traffic system. Zheng et al. [22] designed a new feedback control strategy considering multiple information before the current vehicle. The control inputs consider both the speed differences and headway differences. Traffic safety was enhanced and traffic jam was suppressed effectively. Tang et al. [23,24] proposed an extended macro traffic flow model accounting for the driver's bounded rationality and a speed guidance model accounting for the driver's bounded rationality at a signalized intersection. The driver's bounded rationality influences the behavior of traffic flow. Moreover, Tang et al. [25,26] investigated the impacts of energy consumption and emissions on the trip cost without late arrival at the equilibrium state and proposed an extended car-following model to study the influences of the driver's bounded rationality on his/her micro driving behavior, and the fuel consumption. Recently, Tang et al. [27] also proposed a car-following model to investigate the impacts of signal light on driving behavior, fuel consumption and emissions during the whole process that each vehicle runs across the intersection. I think that they are interesting. Cheng et al. [28] investigated KdV–Burgers equation in a new continuum model based on full velocity difference model considering anticipation effect. The above investigations are helpful in the development of traffic flow theory and they mainly focused on some traffic flow models related to cooperative driving, intelligent transportation system, and feedback control schemes so on. The discrete traffic flow model and its control scheme was few discussed. In this paper, we will study the discrete car-following model.

The remainders of this paper are organized as follows. In Section 2, the discrete car-following model and impulse transfer function were presented. In Section 3, A Lemma was given as a criterion to judge the stability of uncontrolled car-following system. In Section 4, feedback control scheme was designed and a theorem was proposed for controlled system. In Section 5, numerical simulations were carried out in two situations. Some new results were exhibited with figures. In Section 6, the summary is given.

2. Model

Assume that there are N vehicles in a car-following system. All vehicles distribute on a single lane ring road with average density $\rho = 1/h$, where h is the average headway. Fig. 1 shows the illustration of the car-following system where the nth vehicle follows the $(n+1)$th vehicle. According to Newell's model, the following vehicle's motion equation is given as

$$\frac{dx_n(t+T)}{dt} = F(\Delta x_n(t)) \tag{1}$$

where T is an adjusting time of driver. $\Delta x_n(t) = x_{n+1}(t) - x_n(t)$ and $x_n(t)$ denote the headway and position of the nth vehicle at time t respectively. The idea is that the driver adjust the velocity of the nth vehicle at time t according to the headway to reach the optimal velocity $F(\Delta x_n(t))$ after a time T.

The differential equation is discretized with the following equation

$$\frac{x_n(t+T+\Delta t) - x_n(t+T)}{\Delta t} = F(\Delta x_n(t)) \tag{2}$$

Let $\Delta t = T$ then one can obtain the following difference equation

$$x_n(t+2T) = x_n(t+T) + F(\Delta x_n(t)) * T \tag{3}$$