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Soret and Dufour effects on three dimensional Oldroyd-B fluid

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ABSTRACT

In this article, a realistic three dimensional magnetohydrodynamic flow of an Oldroyd-B fluid is studied after developing a convergent analytic scheme. Soret and Dufour effects with mixed convection are taken into account. The governing highly non-linear partial differential equations are transformed into the system of ordinary differential equations using similarity transformations. The resulting problems are computed by homotopy analysis method (HAM). Profiles of dimensionless velocities, temperature and concentration are plotted and discussed for various emerging physical parameters. Numerical values of physical quantities of interest such as local Nusselt number and local Sherwood number are tabulated. A comparative study with existing literature are found in an excellent agreement.

1. Introduction

The non-Newtonian fluids are quite popular among the recent researchers because of their occurrence in industries, biology and engineering. Examples of such fluids include polymer solutions, concrete/china clay, foams and emulsions, sewage sludge, pharmaceutical chemicals, cosmetics, paints and biological fluids such as blood, synovial fluid, salvia etc. Due to different rheological characteristics the non-Newtonian fluids cannot be described by a single constitutive equation. Therefore, several constitutive equations are presented in the literature via three main categories namely the differential, the rate and integral types. Existing literature witnesses that much attention is focused to the one and two-dimensional flows of differential type fluids. Some investigations are also made for the one and two-dimensional flows of rate type fluids. Researcher feel scarcity of information on the three-dimensional flows of subclass of rate type fluids. For example, Fetecau et al. [1] discussed the unsteady flow of an Oldroyd-B fluid induced by a constantly accelerating plate between two side walls perpendicular to the plate. They solved the problem using Fourier Sine transform. It is found that the side walls have a significant influence on the fluid motion for larger times. Hydromagnetic Couette flow of an Oldroyd-B fluid in a rotating system has been investigated by Hayat et al. [2]. They consider the motion of conducting fluid bounded by the parallel plates. They have analyzed the effects of Coriolis force as well. Jamil et al. [3] investigated the unsteady helical flows of an Oldroyd-B fluid. The oscillating motion of an Oldroyd-B fluid between in two infinite circular cylinders is studied by Fetecau et al. [4]. It is concluded that the amplitude of transient oscillations is smaller in magnitude for the case of Oldroyd-B fluid when compared with the Newtonian fluid. Tong et al. [5] presented the unsteady helical flows of generalized Oldroyd-B fluid. They have extended the work of Ref. [3] for the generalized Oldroyd-B fluid. Zheng et al. [6] found the exact solutions for flow of

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generalized Oldroyd-B fluid due to an infinite accelerating plate. They analyzed the magnetohydrodynamic effects. Hayat and Alsaedi [7] presented the thermal radiation and Joule heating effects in MHD flow of an Oldroyd-B fluid with thermophoresis. Global results for Oldroyd-B fluids in exterior domains have been developed by Hieber et al. [8]. Similar solution for three-dimensional flow in an Oldroyd-B fluid over a stretching sheet has been examined by Hayat et al. [9]. Stability of thermal convection of an Oldroyd-B fluid in a porous medium with Newtonian heating is analyzed by Niu et al. [10]. Hayat et al. [11] presented the mixed convection three-dimensional flow of an upper-convected Maxwell (UCM) fluid subject to magnetic field and diffusion-thermo effects.

The flow induced by a stretching surface appears in several engineering processes including in the paper and glass production, extrusion of plastic and fiber sheets, wire coating and metal spinning etc. Since the seminal work of Crane [12] on stretched flows, several researchers ([13–16] and many refs. therein) have studied such flows under different aspects. Effects of heat and mass transfer on the flows caused by a stretching surface are also explored. For-instance Shahzad et al. [17] presented the simultaneous effects of thermal boundary layer on time-dependent flow over a stretching surface through permeable medium. Pal and Chartterjee [18] presented the heat and mass transport process in magnetohydrodynamic non-Darcian flow of a micropolar fluid over a stretching sheet embedded in a porous medium with non-uniform heat source and thermal radiation. Hayat et al. [19] presented the heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid. Pal et al. [21] studied the effects of Soret Dufour, chemical reaction and thermal radiation on unsteady MHD flow with mixed convective over a stretching sheet. The Group theory transformation for Soret and Dufour effects on free convective heat and mass transfer with thermo chemical reaction over a porous stretching surface in the presence of heat source/sink with other physical properties are found in the literature e.g., see [19–21].

To our knowledge, Soret and Dufour effects on the MHD three-dimensional flow of an Oldroyd-B fluid over a stretching surface are not investigated yet. Even such attempt without Soret and Dufour effects is not presented. Hence, the present article include the mathematical formulation for such flows. The involved nonlinear analysis is computed using homotopy analysis method (HAM) [22–27]. The results are also compared with existing literature for the limiting solutions. Influence of pertinent parameters are studied in detail.

2. Formulation of the problem

Here, we study the incompressible three-dimensional flow of an Oldroyd-B fluid past a stretching surface. The fluid occupies the space $z > 0$. Heat and mass transfer effects are considered in the presence of Soret and Dufour effects. The fluid is electrically conducting in the presence of an applied magnetic field B_0. The velocity, temperature and concentration fields are governed by the following three-dimensional boundary layer equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$\begin{aligned}
&u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+\lambda_1\begin{pmatrix} u^2\dfrac{\partial^2 u}{\partial x^2}+v^2\dfrac{\partial^2 u}{\partial y^2}+w^2\dfrac{\partial^2 u}{\partial z^2} \\ +2uv\dfrac{\partial^2 u}{\partial x\partial y}+2vw\dfrac{\partial^2 u}{\partial y\partial z}+2uw\dfrac{\partial^2 u}{\partial x\partial z}\end{pmatrix}\\
&\quad=\nu\left\{\frac{\partial^2 u}{\partial z^2}+\lambda_2\begin{pmatrix} u\dfrac{\partial^3 u}{\partial x\partial z^2}+v\dfrac{\partial^3 u}{\partial y\partial z^2}+w\dfrac{\partial^3 u}{\partial z^3} \\ -\dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial z^2}-\dfrac{\partial u}{\partial y}\dfrac{\partial^2 v}{\partial z^2}-\dfrac{\partial u}{\partial z}\dfrac{\partial^2 w}{\partial z^2}\end{pmatrix}\right\}\\
&\quad-\frac{\sigma B_0^2}{\rho}\left(u+\lambda_1 w\frac{\partial u}{\partial z}\right)+g^*[\beta_T(T-T_\infty)+\beta_C(C-C_\infty)]
\end{aligned} \tag{2}$$

$$\begin{aligned}
&u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+\lambda_1\begin{pmatrix} u^2\dfrac{\partial^2 v}{\partial x^2}+v^2\dfrac{\partial^2 v}{\partial y^2}+w^2\dfrac{\partial^2 v}{\partial z^2} \\ +2uv\dfrac{\partial^2 v}{\partial x\partial y}+2vw\dfrac{\partial^2 v}{\partial y\partial z}+2uw\dfrac{\partial^2 v}{\partial x\partial z}\end{pmatrix}\\
&\quad=\nu\left\{\frac{\partial^2 v}{\partial z^2}+\lambda_2\begin{pmatrix} u\dfrac{\partial^3 v}{\partial x\partial z^2}+v\dfrac{\partial^3 v}{\partial y\partial z^2}+w\dfrac{\partial^3 v}{\partial z^3} \\ -\dfrac{\partial v}{\partial x}\dfrac{\partial^2 u}{\partial z^2}-\dfrac{\partial v}{\partial y}\dfrac{\partial^2 v}{\partial z^2}-\dfrac{\partial v}{\partial z}\dfrac{\partial^2 w}{\partial z^2}\end{pmatrix}\right\}\\
&\quad-\frac{\sigma B_0^2}{\rho}\left(v+\lambda_1 w\frac{\partial v}{\partial z}\right),
\end{aligned} \tag{3}$$