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A generic signature of a fluctuating magnetic field: An additional turnover prior to the Kramers' one

Shrabani Mondal, Alendu Baura, Sudip Das, Bidhan Chandra Bag *

Department of Chemistry, Visva-Bharati, Santiniketan 731 235, India

HIGHLIGHTS

- A new turnover prior to the well known Kramers' one may appear.
- The new turnover phenomenon is a generic signature of the fluctuating magnetic field.
- The location of the turnover is shifted towards right in the presence of non-Markovian thermal bath.
- Both the turnovers may disappear at high strength of magnetic field.

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ABSTRACT

In this paper we have presented the dynamics of a Brownian particle which is coupled to a thermal bath in the presence of a fluctuating magnetic field (FMF). By virtue of the FMF the Brownian particle experiences a time dependent damping strength for the $x-$direction motion even in the presence of a stationary Markovian thermal bath. There is a transition state along this direction. The time dependent damping strength leads to appear a bi-turnover phenomenon in the variation of the barrier crossing rate as a function of the thermal bath induced damping strength. It is a generic signature of the fluctuating magnetic field.

1. Introduction

Study of the ion conducting electrolytic materials is a very important area in physics and chemistry in the recent past. The materials have potential applications in a diverse range of all-solid-state devices, such as rechargeable lithium batteries, flexible electrochromic displays and smart windows [1]. The properties of the electrolytes are tuned by varying chemical composition to a large extent and hence are adapted to specific needs [2]. High ionic conductivity is needed for optimizing the glassy electrolytes in various applications. It would be very interesting if one can tune the ionic conductivity according to a specific need by a physical method. In the very recent studies [3–5], it has been shown that the conductivity of an electrolytic material can be tuned by an applied magnetic field. In this context one may use an electrical magnet. Then the applied voltage may be random in nature. Because of that the current induced magnetic field may be fluctuating one. The effect of the colored Gaussian random magnetic field on the barrier crossing dynamics of a particle has been studied in Ref. [6]. The fluctuating magnetic field introduces unusual type multiplicative noises. Cross coordinate as well as velocity dependent multiplicative noises may appear in the presence of a fluctuating magnetic field. By virtue of the fluctuating field, the Brownian particle experiences a time dependent damping strength for the x-direction motion even in the presence of

* Corresponding author.
E-mail address: bidhanchandra.bag@visva-bharati.ac.in (B.C. Bag).

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a stationary Markovian thermal bath. Then study of the barrier crossing rate constant as a function of damping strength (due to thermal bath) might be an important issue. Recent studies on the barrier crossing dynamics [3–8] and the Kramers' turnover [9–15] imply that these issues are still active research areas in the field, non equilibrium statistical mechanics. Keeping it in mind we have investigated the barrier crossing dynamics of a charged particle in the presence of a fluctuating magnetic field which is Gaussian in characteristic. Our investigation reveals that a bi-turnover phenomenon may appear in the variation of the barrier crossing rate constant as a function of the thermal bath induced damping strength. Thus a new turnover appears in addition to the Kramers' one [9–22]. The existence of the turnovers depends on the strength of both the constant and the fluctuating magnetic fields.

Before leaving this section we should mention that the present study is based on the diffusive behavior of the charged particle in the presence of a magnetic field. It may be relevant in the field of diffusion of plasma and cosmic rays. To be noted here that investigation of the effect of the magnetic field on the diffusion of plasma [23] and the cosmic rays [24] is now a subject of key interest. Recently, study of the motion of charged particle in electrolytic solution is getting attention [25,26]. Response to a periodic magnetic field by a charged particle has been investigated in Ref. [5,6,27]. The Fokker–Planck description and the related aspects for the Brownian motion of ion in the presence of a magnetic field and the non-Markovian thermal bath are the issues of current interest [4,28,29]. In other areas also study of motion of charged particle in the presence of a magnetic field is getting attention in the recent past [30,31].

The outline of the paper is as follows: In Section 2 we have presented the model. The signature of the fluctuating magnetic fields on the barrier crossing dynamics has been explored in Section 3. The paper is concluded in Section 4.

2. The model

In the present study we have considered the Brownian motion of a charged particle which is bi-linearly [32–34] coupled to the modes of the thermal bath in the presence of a magnetic field. The Brownian motion can be described with the following system-bath Hamiltonian with SI unit,

$$\begin{aligned} H = {} & \frac{[p_x - qA_x]^2}{2m} + \frac{[p_y - qA_y]^2}{2m} + \frac{[p_z - qA_z]^2}{2m} + V(x, y, z) \\ & + \sum_i \left[\frac{p_{ix}^2}{2m_i} + \frac{p_{iy}^2}{2m_i} + \frac{p_{iz}^2}{2m_i} + \frac{m_i}{2}\left(\omega_i x_i - \frac{c_i}{m_i\omega_i}x\right)^2 \right] \\ & + \sum_i \left[\frac{m_i}{2}\left(\omega_i y_i - \frac{c_i}{m_i\omega_i}y\right)^2 + \frac{m_i}{2}\left(\omega_i z_i - \frac{c_i}{m_i\omega_i}z\right)^2 \right] \quad . \end{aligned} \tag{1}$$

Here $p_j (j = x, y, z)$ are components of the canonical momentum vector of the Brownian particle with mass m and charge q. The above Hamiltonian as well as the Lagrangian for the charged particle suggests the following relation between the components of canonical momentum vector and the vector potential [35]

$$p_j = mu_j + qA_j \quad . \tag{2}$$

Here we have denoted components of velocity of the particle by u_j. The particle experiences a vector potential with components A_j due to the presence of a magnetic field, **B**. In addition to the vector potential, the above Hamiltonian contains a scalar potential energy function, $V(x, y, z)$. We now consider the remaining part of the above Hamiltonian which is related to the thermal bath and the interaction between the system and the bath. p_{ix}, p_{iy} and p_{iz} in the Hamiltonian are components of the momentum vector of the ith harmonic bath mode having frequency, ω_i and mass, m_i. The components of the position vector of this bath mode are x_i, y_i and z_i, respectively. The strength of the coupling between the system and the ith bath mode is c_i. To be noted here that the bath modes are considered as isotropic harmonic oscillators. The relevant equations of motion corresponding to the above Hamiltonian can be written as

$$\dot{x} = \frac{1}{m}(p_x - qA_x) \ , \tag{3}$$

$$\dot{y} = \frac{1}{m}(p_y - qA_y) \ , \tag{4}$$

$$\dot{z} = \frac{1}{m}(p_z - qA_z) \ , \tag{5}$$

$$\begin{aligned} \dot{p_x} = {} & \frac{q}{m}(p_x - qA_x)\frac{\partial A_x}{\partial x} + \frac{q}{m}(p_y - qA_y)\frac{\partial A_y}{\partial x} + \frac{q}{m}(p_z - qA_z)\frac{\partial A_z}{\partial x} - \frac{\partial V}{\partial x} \\ & + \sum_i c_i x_i - x\sum_i \frac{c_i^2}{m_i\omega_i^2} \ , \end{aligned} \tag{6}$$