The price-volume relationship caused by asset allocation based on Kelly criterion

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H I G H L I G H T S

- We show that the price-volume relationship could be explained by investors' adoption of Kelly strategy.
- The accumulated absolute deviation of the risk asset's return from the risk-free rate is positively related to trading volume.
- In a multi-asset scenario, one asset's trading volume could be effected by other assets' price change.
- We find evidence of such relationship from both Chinese and American stock markets.

A R T I C L E   I N F O

A B S T R A C T

It is noticed that there is a relation between assets' return and trade volume in financial markets, but existing theory could not explain how exactly they are connected or what the relation is in a general scenario. Based on the hypothesis that investors who adopt a Kelly trading strategy will adjust their position periodically, we present a model describing the explicit price-volume relation. The model shows that factors related with the volume of trade are: (1) the total volume of the risk asset; (2) the optimal proportion of the risk asset implied by the Kelly criterion; and (3) the accumulated absolute deviation of the risk asset's return from the risk-free rate. In a multi-asset scenario, the factor (2) and (3) of one asset could partly explain other assets' trading volume. Empirical test with data from the Chinese and the U.S. stock markets verifies such relation.

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1. Introduction

The price-volume relation in financial markets has been focused for a long time. For at least four reasons, such relation is important [1]. First, it provides information about the structure of financial markets [2]. Second, such relation is important for studies that use both price and volume data [3]. Third, it helps to explain the fat-tail distribution of investment funds' return rates. And forth, price-volume relations have significant implications for determining the number of hedging contracts in futures contracts [4]. Researchers try to make use of the price-volume relation, e.g., to evaluate the performance of traders [3], to utilize more information [5], or to optimize the execution of trading orders [6].

Recent studies concerning the price-volume relationship are mainly empirical work, dedicated to verifying such relation for different markets from different perspectives, e.g., focusing on security markets or gold markets, employing different paradigms such as econometric methods or simulation, using different measurements of price changes and trading volume, and choosing different sample period. For example, previous researches concerning the price-volume relationship include the influence of market structure in stock markets [7,8], in a continuous double auction market [2], the empirical test for the

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price-volume relationship in housing markets [9], in gold markets [10] and in future markets [4], the relationship between absolute values of daily price changes and daily trading volume of the market indices [11], between the square of price changes and periodic trading volume from the daily data of the cotton futures [12], and empirical tests employing different levels of data, from yearly data of individual common stocks [13] to two-month intervals data of forward contracts [14], as well as trade level data [8,15,16].

Two kinds of theories are proposed to explain why the positive relation between price change and trade volume holds. The first theory employ the mixture of distribution hypothesis (MDH) [12,17], saying that the structure of investors is the key factor in affecting the price-volume relation. The other theory is based on the asymmetric information hypothesis (AIH) [18,19], concluding that the correlation between price change and trade volume is determined by the information flow. Both the MDH and the AIH could give the reason of the price-volume relation, but it needs more assumption to give an explicit description of what the relation is.

In this paper, we build a model which gives an explicit equation describing the relation between price change and the amount of trading volume, which could not be found in existing literatures or given by existing hypothesis explaining the price-volume relationship. Recent empirical studies focus on verifying the existence of the relationship in different samples [10,20–22], but lack explanation for why the relation holds. The MDH and the AIH could be tested with agent-based simulating method [2,23], but not with data from real financial markets. The relation implied by our model, is supported by data from the Chinese and U.S. stock markets.

The model we build is based on the hypothesis that investors adjust their portfolios based on the Kelly criterion [24], which means that investors should optimize the size of bets for each period rather than follow a buy-and-hold strategy [25]. The rationality of adopting Kelly strategy has been throughly illustrated by Hakansson [26] and Merton [27] etc., giving that it could maximize the geometric average return rather than the arithmetic average return. The prior studies give our model a solid foundation.

The rest of the paper is organized as follows: Section 2 describes the basic model with a single risk asset and provides some discussions; Section 3 describes the generalized model with multiple risk assets, and discusses about the spillover effect in a two risk assets setting; Section 4 provides empirical evidence for the model with data from the Chinese and the U.S. stock markets. Section 5 contains the concluding remarks.

2. The Kelly strategy and the price-volume relation in a single risk asset scenario

In this section, we introduce the Kelly strategy and the symbols used for building the model. The price-volume equation is derived in a single risk asset scenario, which reveals main elements related with trading volume.

2.1. The Kelly strategy

Assume that investors who adopt Kelly strategy face two kinds of trading opportunities at each time: one is a risk asset, such as openly traded stocks or bonds issued by private companies; the other is a risk-free asset such as the government debt. Write $r_i \in (-\infty, \infty)$ as the continuous compound risk return in time i, whose density function is written as $f(\cdot)$. Which is subjected to $f(r) \geq 0, \forall r_i \in (-\infty, \infty)$ and $\int_{-\infty}^{\infty} f(r)\,dr = 1$. $r_f > 0$ is the risk-free continuous compound interest rate which should be an constant. Assume that at the beginning time 0 the Kelly-type investors have a total wealth $W_0 = 1$, and write W_i as the total wealth of investors at the beginning of time i. Write θ as the proportion of risk assets hold by each investor. Because the investments are made according to a Kelly criterion, θ should be constant and W_i follows:

$$W_i = W_{i-1}(\theta e^{r_i} + (1 - \theta)e^{r_f}). \tag{1}$$

And the average continuous compound return AR after N period should be:

$$AR = \frac{1}{N}\sum_{i=1}^{N} \ln(\frac{W_i}{W_{i-1}}) = \frac{1}{N}\sum_{i=1}^{N} \ln(\theta e^{r_i} + (1 - \theta)e^{r_f}). \tag{2}$$

As $N \to \infty$, we have the expected average continuous compound return as:

$$E(AR) = E(\ln(\theta e^{r_i} + (1 - \theta)e^{r_f})). \tag{3}$$

The investors want to maximize their average continuous compound return by choosing θ, and the problem could be formalized as:

$$\max_{\theta} E(\ln(\theta e^{r_i} + (1 - \theta)e^{r_f})). \tag{4}$$

The existence and uniqueness of θ is promised by previous literatures in Merton [27] and Haugh [25].