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Applying Greek letters to robust option price modeling by binomial-tree

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HIGHLIGHTS

- A new model is proposed for pricing a European option is presented.
- Binomial tree method is combined with the Greek letters.
- The proposed model is shown to be superior compared with previous models.

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ABSTRACT

In this paper, a new model is proposed for pricing a European option using the binomial tree method in conjunction with the Greek letters. In the proposed method, the covariance matrix of high and low stock prices was calculated in an uncertainty region. Applying robust option pricing model, an 'interval' of prices (instead of 'spot' prices) for an option was obtained. Greek letters were incorporated into a robust option model to ameliorate the accuracy of the interval price. It was found out that the interval prices obtained by the present model were flexible with increased accuracy compared with those obtained by the robust option using the binomial tree model. It is also indicated that the advantage of the present model over existing models is more tangible in the event of 'out of the money' call option. Furthermore, the accuracy improvement was found to be less noticeable when the maximum costs were equal to each other.

1. Introduction

An option is an important derivative, hedging the risk of price changes in the future. Among the various derivative instruments, option trading gives the holder the right and not the obligation to buy or sell an underlying asset at a predetermined price at a specified future time [1]. As the stock prices vary in time, the investors' losses and benefits are a function of option pricing. Today, the option literature knows how to price options on shares, bonds, foreign currency, futures, options, commodities and derivative assets. Today, European countries, American countries and Asian countries use option pricing in their economical chains. For example, the pricing of European options on two underlying assets with delays are studied by Lin et al. [2]. Chen et al. [3] analytically studied and modeled an American option prices and supported their solution with numerical results. Numerical and empirical experiments are conducted by Ma et al. [4] to propose an accurate European option pricing model under Fractional Stable Process. Assets of Asian rainbow options pricing are explored by Wang et al. [5].

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Thus far, various option models have been developed relative to very different assets. There are two main categories of option models; models that presume continuous trading, such as Black and Scholes [6] and Merton [7] models, and those presuming trading at discrete time intervals such as binomial-tree model. There are also three pricing methods; analytical, numerical and empirical approaches. Each of them could be applied for American or European options. The present section does not consider the vast literature on all option valuation models neither all option types. Instead, its stress is on binomial-tree model, that is a numerical discrete-in-time approach for the valuation of European option. As additional tools for the improvement of option price modeling, consideration is further given to the history of robust modeling and Greek letters in subsequent parts of the present section.

The principles of risk-neutrality and riskless portfolios in Black and Scholes Model (BSM), established the basis for the theory of option pricing. However, its employed mathematical tools might have appeared too advanced, academic or even awkward, as they have tended to obscure the underlying economics. This issue attracted various economists to seek for more easily accessible models that maintain the economically relevant properties of the BSM model. Among them, the numerical discrete-time binomial models are the most well-known modeling approach. The binomial approach to option pricing grew out of the papers published by Cox et al. [8] and Rendleman and Bartter [9]. Using a backward induction algorithm, the CRR model by Cox et al. [8] and RB model by Rendleman and Bartter [9] were found to be an approximation to a Brownian motion and as the period length tends to zero, the sequence of corresponding binomial models converges weakly to a geometric Brownian motion, that underlies the BSM model. However, Rendleman and Bartter [9] indicated that the RB model requires less operation counts for backward induction than the CRR model due to the symmetry in probabilities. Although the CRR and BR binomial models are based on an efficient backward induction algorithm, they suffer from various limitations and drawbacks in practical applications, that attracted the many researches to modify their models. For example, the convergence improvement and the discretization errors involved in binomial models are studied by Jarrow and Rudd [10], Boyle [11], Omberg [12], Tian [13], Leisen and Reimer [14], Chang and Palmer [15], Dai et al. [16], Joshi [17], Muroi and Yamada [18]. In the JR model by Jarrow and Rudd [10], a binomial model where the first two moments of the discrete and continuous time log return processes match, was constructed. The trinomial approaches suggested by Boyle [11] and Tian [13] are shown to improve the convergence speed. Omberg [12] applied the technique of Gauss–Hermite and presented a new model using the quadrature as a solution to the backward recursive integration problem. Leisen and Reimer [14] applied an odd number of periods with the tree centered around the strike value of interest and claimed that, on average, the same degree of accuracy is achieved 1400 times faster with the new binomial models. For the model suggested by Chang and Palmer [15], the CP tree, the nodes in the tree are moved only a small distance so that the strike falls onto the geometric average of the two neighboring nodes. A new family of binomial trees as approximations to the Black–Scholes model with higher order of convergence are introduced by Joshi [17]. Recently, new computational algorithms based on the spectral expansion method are introduced by Muroi and Yamada [18] and were found to be fast algorithms to evaluate the price of double barrier options using binomial trees. Another group of modifications to binomial models can be found in the articles published by Amin [19], Jiang and Dai [20], Qian et al. [21] and Kim and Qian [22] who developed jump diffusion models in binomial-tree approach. Further modifications and applications of binomial models belong to the generalization of the m-dimensional binomial trees for the multi-asset options [23–29], stock options with discrete dividends [30,31], non-standard options [32–34], calculating the upper and lower bounds [35] and robust approach [36].

The two methods (BS and binomial tree method) have their own drawbacks. For instance, to calculate the selection cost, these methods predict only a spot price or an interval price with complexity, low accuracy and low flexibility. Consequently, novel solutions have been recently introduced to improve these two methods. For example, Bandi and Bertsimas [37] applied robust optimization to model option pricing in such a way that minimized the worst example of replication error. A significant privilege of this model was its flexibility as it modeled different levels of investor risk aversion and different choices. Hanafizadeh et al. [36] employed robust option pricing along with the binomial tree method. They argued that the complicated calculations needed to anticipate an interval for option price could become redundant by using robust option pricing through binomial tree method while the flexibility of the model (considering the sort of investors' risk aversion and different options) is still preserved. Nevertheless, it is noteworthy that the accuracy of the interval price provided by their model is not sufficiently high, so there is room for further modification. One way to promote the accuracy of their model is to apply robust option pricing along with Greek letters.

Greek letters are the derivatives of the BS model used for hedging risk of option pricing for investors. Thus, integrating the Greek letters with the robust option pricing approach may contribute to a more precise price interval. Greek letters are applied by many researchers. For example, Chen et al. [38] defined Greek letters as the sensitivities of the option price to a single-unit change in the value of a state variable or parameter and derived them for call and put options prices on both dividends and non-dividends paying stocks. They indicated that Greek letters could represent different dimensions of risk and their examination is an efficient method for managing the investors' risk. Papahristodoulou [39], instead of deciding in advance the most appropriate hedging option strategy, presented an LP problem which is formulated by considering all significant Greek parameters of the Black–Scholes formula, such as delta, gamma, theta, rho and kappa and the optimal strategy to select will be simply decided by the solution of that model. The advantage of the model is that it can provide a quick and relatively simple solution in computation. But, it is not necessary to keep the risk exposure on all the Greek letters covered. Hull [1] argues that a zero gamma and a zero vega (or kappa) are less easy to achieve in the concrete operation. Gao [40] proposes a general linear programming model with risk bounds on all the Greek letters. As the risk bounds are enlarged in the model, the optimum return increases. So, a risk-return trade-off can be done for an investor by setting different risk