Physica A 505 (2018) 398–412

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

A simple analytics framework for evaluating mean escape time in different term structures with stochastic volatility

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HIGHLIGHTS

- Mean escape time of interest rate product with stochastic volatility is analyzed.
- Different scenarios of term structures are simulated for theoretical and empirical yield curves.
- Survival probability rapidly decays in high volatility and adjacency to barriers.
- Change in long-term maturity produces the largest variation of mean escape time.
- Noise enhanced stability phenomenon is detected in flat yield, T-bill, and Eurobond.

ARTICLE INFO

Article history:
Received 13 July 2017
Received in revised form 30 January 2018
Available online 7 March 2018

Keywords:
Mean escape time
Interest rate
Term structures
Stochastic volatility
Noise enhanced stability

ABSTRACT

The aim of this paper is to propose a simple framework for analyzing the mean escape time of interest rate product in different term structures with stochastic volatility. In the modeling perspective, we utilize the one factor Hull–White model to design the dynamics of interest rate returns whose stochastic volatility term is assumed to follow the Cox–Ingersoll–Ross process. Furthermore, we apply the Nelson–Siegel function to simulate various scenarios of term structure based on the US Treasury bill and European bond. Then, we analyze the mean escape time surfaces of different term structures for theoretical flat yield curves and two empirical S-shaped yield curves whose term structures of short-, mid-, and long-term maturities are shifted by the modified parameters of the Nelson–Siegel function. We observe that the survival probability begins with one and reduces to zero with different rate of decay for different interest rates and volatilities. Furthermore, the results of empirical term structures imply that the adjustment in the yield of long-term maturity changes the structure of MET surface more significantly than that of short- and long-term maturities. Lastly, we detect the noise enhanced stability phenomenon from the mean escape time for all cases of flat yield, Treasury bill, and European bond.

1. Introduction

Financial system is renowned for its nature of complexity [1]. Not surprisingly, a financial market is driven by various participants using different investment philosophies and strategies including fundamentalist, arbitragers, speculators, and noise traders. Consequently, the non-linear interactions among them yield the presence of natural randomness [2]. In this context, the recent studies in Econophysics have analyzed the financial market as a single system and its risk based on

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https://doi.org/10.1016/j.physa.2018.02.187
0378-4371/

various network models which leads to discover the correlated phenomena among the agents [3–12]. Econophysics is an interdisciplinary science which applies the concepts of physics to solve the problems in economics and finance [13,14]. Especially, it focuses on the properties of the financial time series in the perspective of the non-Gaussian distributions of returns [15–20] and the multifractal behavior [21–30].

In the modeling perspective, Bachelier [31] approximates the dynamics of a stock based on the random walk process, which fails to capture the stochastic nature of the volatility of financial time series. Accordingly, the advanced model has been proposed to replicate the stochastic volatility based on two-dimensional diffusion processes [32]. Furthermore, the time series of a financial product with barriers can be modeled based on various stochastic differential equations with a specified boundary condition [33]. For instance, if we define the moment when a randomly moving particle touches its boundary refers to a pre-specified risk of a financial product, then the time elapsed until the particle touches the boundary from its starting point becomes a critical information in the perspective of risk management. Therefore, a mean escape time (MET), an average of such elapsed time, can be utilized for analyzing the risk of financial product [34–36].

Although many stylized facts regarding the MET and stochastic volatility have been discovered for the stock markets [37–44], the price dynamics of interest rate products has not received much attention. That is why, in this paper, we focus on developing a simple analytics framework for evaluating the MET of interest rate with respect to different term structures with stochastic volatility. Given that the dynamics and structure of interest rate product differ from those of stock, the computation of MET requires the distinct physical models regarding the diffusion processes. Particularly, two models are utilized to compute the MET of interest rate products as defined in Section 2. The first model is related to the dynamics of interest rate; the most renowned model is the Hull–White model [45]. Despite its novelty, the weakness of the model is its constant volatility term, whose dynamics is known to follow a random process [46]. Thus, we exploit the one factor Hull–White model by consolidating the Cox–Ingersoll–Ross process to integrate the stochastic volatility instead of the constant one [47]. As stated in [32], the stochastic volatility is more realistic approach for analyzing the risk of interest rate derivatives. The second model is related to the shape of term structure, practically known as an yield curve. The term structure is a unique feature of interest rate whose shape presumes the path of the interest rate dynamics in near and distant future. We utilize the Nelson–Siegel function so that its estimated parameters can be served as a ground term structure.

Based on two physical models, we configure the MET of interest rate in 3-dimensional surface where its general trend can be visually identified. As illustrated in Section 3, the 3-months US treasury bill and 1-year Eurobond are used to calibrate the Nelson–Siegel function. Then, we derive different scenarios of term structures by shifting the parameters of the Nelson–Siegel function and discuss the results and economic implications in Section 4. To sum up, our framework provides an accessible application for analyzing the risk of interest rate products, whose barrier condition is known, with flexible term structures while an interest rate at zero maturity is unknown and only can be estimated.

2. Models and methods

2.1. Hull–White model with stochastic volatility

Let $r(t)$ be the time-series of an interest rate with respect to time, t, then the stochastic process of interest rate series can be defined based on the Hull–White model such that,

$$dr(t) = (\theta(t) - ar(t))dt + \sqrt{\sigma^2(t)}dW_1(t) \tag{1}$$

where the volatility, σ^2, is considered to follow the mean reverting stochastic process,

$$d\sigma^2(t) = -\alpha(\sigma^2(t) - m^2)dt + k\sqrt{\sigma^2(t)}dW_2(t). \tag{2}$$

Note that a and $\theta(t)$ in Eq. (1) denote the reversion rate and deterministic function of term structure, respectively. In general, the price of an asset is strongly determined by supplies and demands in the market (e.g. stock, bond, and et cetera). However, the interest rate is highly depending on the economic policy in standard interest rate and many macro-economic conditions, which impose difficulty to consolidate them into a parsimonious model. In this study, for simplicity, we assume that the correlation between $dW_1(t)$ and $dW_2(t)$ be zero, and no other variable is considered for the feedback effect. In addition, we set $\sigma^2(t)$ be non-negative. θ in Eq. (1) can be derived from the current initial term structure as follows.

$$\theta(t) = \frac{\partial F(0,t)}{\partial t} + aF(0,t) + \frac{\sigma^2}{2a}(1 - \exp(-2at)). \tag{3}$$

Since $F(t_1, t_2)$ represent the forward interest rate during (t_1, t_2), $F(0, t)$ is the instantaneous forward rate from present to t. The derivative of the instantaneous forward rate with respect to t can be approximated as follows.

$$\left[\frac{\partial F(0,t)}{\partial t}\right]_{t=t'} \approx \frac{f(0, t' + \Delta t) - -f(0, t')}{\Delta t}. \tag{4}$$

Furthermore, α, m^2, and k in Eq. (2) denote the mean reversion rate of $\sigma^2(t)$, long-run variance of σ^2, and volatility of σ^2, respectively. Then, the transition probability, which refers to the time-series of interest rate at t (begins with r_0 and σ_0^2) depending on r and σ^2, should be defined.