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Superstable cycles and magnetization plateaus for antiferromagnetic spin-1 Ising and Ising–Heisenberg models on diamond chains

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HIGHLIGHTS

- Superstable points correspond to a ground state phase (the none-magnetic phase).
- Superstable points are connected to zero magnetization plateau.
- The first maximum Lyapunov exponent plateau is rather similar to the second plateau of magnetization if there exists a superstable point.
- The data suggests a non-trivial relation between ground states and stable fixed.

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ABSTRACT

We consider the appearance of superstable cycles in the dynamical approach to the anti-ferromagnetic and ferromagnetic spin-1 Ising and Ising–Heisenberg models on diamond chains, and their connection with magnetization plateaus. The rational mappings, which provide the statistical properties of the model, are derived by using recurrence relations technique. We consider stability properties of the mapping, providing evidences of a connection between magnetization plateaus and dynamical properties, as the behavior of Lyapunov exponents. The newfound correspondence between superstable point and zero magnetic plateau in spin-1 models on a diamond chain has been tested for a range of parameters.

1. Introduction

The theory of dynamical systems, besides its intrinsic theoretical relevance, recently had a significant impact on a wide range of disciplines from physics to ecology and economics, providing methodological tools which are currently employed in financial and economic forecasting, environmental modeling, medical diagnosis, industrial equipment diagnosis and so on [1,2]. An important physical setting in which such dynamical techniques are profitably applied is equilibrium statistical mechanics of lattice models, more specifically in the investigation of physical properties of low-dimensional classic and quantum spin systems in an external magnetic field.

In this paper we apply dynamical techniques in the analysis of Ising and Ising–Heisenberg spin models on a diamond chain, which are of current interest for a number of reasons. As a matter of fact, these models can be solved exactly by

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using different mathematical methods, and they exhibit a wide range of interesting properties such as the appearance of intermediate plateaus in the magnetization curves, geometric spin frustration, multiple peak structure of the magnetic susceptibility and specific heat [3–13].

The dynamical systems approach [14,15] has been used in many different physical situations, deepening our understanding of the phase structure and critical properties of spin and gauge models: it is particularly powerful in the analysis of exact solution of spin models on hierarchical lattices, which in many cases accurately approximate real ones (the so called Bethe–Peierls approximation). The specific dynamical method we will employ is the recursion relations method, where the lattice is cut into branches, and the full partition function is expressed in terms of the reduced partition functions of all branches. We remark that by using the term *dynamical* we do not imply the introduction of a temporal evolution on the system yielding in the long time limit the canonical distribution, like in Metropolis or Glauber dynamics (stochastic dynamics) [16], or Nosé–Hoover equations of motion (deterministic dynamics) [17]; as we show in the next section, the dynamical systems we are introducing arise from recurrence relations for the partition functions of the systems we are considering: in this sense the asymptotic features of the dynamical systems induced by recurrence relations are connected to the thermodynamic limit of the physical lattice model. The use of dynamical systems concepts and methods, without any physical time involved, is common to other methods in statistical mechanics, like renormalization group flows (see *e.g.* [18]), though we notice that our dynamical systems operate at fixed values of the coupling constants. From the recursion relations of the partition function we are able to infer the thermodynamic limit of relevant thermodynamics quantities, such as magnetization, magnetic susceptibility and specific heat. Besides yielding thermodynamic averages, the recurrence relations have been employed to investigate other relevant quantities, like Yang Lee zeros of the partition function (see for example [19], for the case of one-dimensional Potts model).

In order to physically motivate our analysis, we remark that spin-1 Ising and Ising–Heisenberg models on diamond chains are a very good approximation for atoms of homometallic magnetic complex $[Ni_3(C_4H_2O_4)_2(\mu_3-OH)_2\,(H_2O)_4]_n\cdot(2H_2O)_n$ [20,21], and the molecular compound $[Ni_8(\mu_3-OH)_4(OMe)_2(O_3PR_1)_2(O_2C^tBu)_6\,(HO_2C^tBu)_8]$ [22]. Magnetic-property measurements on such compounds indicate the coexistence of both antiferromagnetic and ferromagnetic interactions between the magnetic centers, Ni ions with spin 1, which indeed suggests to investigate theoretically the magnetic properties of such compounds. Another related interesting material is $Cu_3(CO_3)_2(OH)_2$ – known as natural azurite (Copper Carbonate Hydroxide) – which can be well described by using the quantum antiferromagnetic Heisenberg model on a generalized diamond chain [23–25]. A remarkable feature of these systems is their exact solvability through recurrence relations techniques; within this method, as we already mentioned, statistical properties of a system are associated to one or multidimensional rational mappings. In the antiferromagnetic case both models exhibit a complex behavior, featuring superstable points and cycles, and magnetic plateaus.

The aim of this paper is to study the dynamical approach, and notably superstability in the above mentioned models [26–30]. We propose a non-trivial connection of superstable points and Lyapunov exponents to magnetic plateaus.

The paper is organized as follows: in the next section we give a brief description of the spin-1 Ising and Ising–Heisenberg models on diamond chains and we describe how the dynamical approach allows a detailed analysis of physical properties. In Section 3 we address stability features of the dynamical mappings we derived for such models, and their connections with magnetization; in addition we compare our results with the experimental data in [20]. Finally, in Section 4, we present our concluding remarks.

2. Models and their dynamic solutions

In this section we describe the quantum spin-1 Ising–Heisenberg model and the classical spin-1 Ising model, both on diamond chain. Since the equations for both models are rather similar we will explicitly consider one case, providing further details in the Appendix. The spin-1 Ising–Heisenberg model on a diamond chain (see Fig. 1) and its classic analogue (spin-1 Ising model on a diamond chain) are defined by the following Hamiltonians, written in terms of block contributions:

$$\begin{aligned}H_{IH} = \sum_i^N H_{IH,i} = \sum_i^N (&J(S^x_{a,i}S^x_{b,i}+S^y_{a,i}S^y_{b,i}+S^z_{a,i}S^z_{b,i})+K(S^x_{a,i}S^x_{b,i}\\&+S^y_{a,i}S^y_{b,i}+S^z_{a,i}S^z_{b,i})^2+J_1(\mu^z_i+\mu^z_{i+1})(S^z_{a,i}+S^z_{b,i})+K_1((\mu^z_i)^2\\&+(\mu^z_{i+1})^2)((S^z_{a,i})^2+(S^z_{b,i})^2)+\Delta((S^z_{a,i})^2+(S^z_{b,i})^2)+\Delta_1\frac{(\mu^z_i)^2+(\mu^z_{i+1})^2}{2}\\&-h_H(S^z_{a,i}+S^z_{b,i})-h_I\frac{\mu^z_i+\mu^z_{i+1}}{2})\end{aligned} \tag{1}$$

and

$$\begin{aligned}H_I = \sum_i^N H_{I,i} = \sum_i^N (&J(S_{a,i}S_{b,i})+K(S_{a,i}S_{b,i})^2+J_1(\mu_i+\mu_{i+1})(S_{a,i}+S_{b,i})\\&+K_1((\mu_i)^2+(\mu_{i+1})^2)((S_{a,i})^2+(S_{b,i})^2)+\Delta((S_{a,i})^2+(S_{b,i})^2)\\&+\Delta_1\frac{(\mu_i)^2+(\mu_{i+1})^2}{2}-h_H(S_{a,i}+S_{b,i})-h_I\frac{\mu_i+\mu_{i+1}}{2}).\end{aligned} \tag{2}$$