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Temporal correlations in the Vicsek model with vectorial noise

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A B S T R A C T

We study the temporal correlations in the evolution of the order parameter $\phi(t)$ for the Vicsek model with vectorial noise by estimating its Hurst exponent H with *detrended fluctuation analysis* (DFA). We present results on this parameter as a function of noise amplitude η introduced in simulations. We also compare with well known order–disorder phase transition for that same noise range. We find that – regardless of detrending degree – H spikes at the known coexistence noise for phase transition, and that this is due to nonstationarities introduced by the transit of the system between two well defined states with lower exponents. We statistically support this claim by successfully synthesizing equivalent cases derived from a transformed fractional Brownian motion (TfBm).

1. Introduction

What we now know as the *Vicsek Model* (VM) is a metric model based on the supposition that each element (without an internal structure) in a swarm has a tendency to mimic the motion of its neighbors, being this interaction affected by an intrinsic noise [1]. Despite its inherent simplicity, the VM presents many outstanding features such as phase transitions, self-organization and fluctuations (many and varied studies based on the VM can be found on reviews [2–4] and cited references). The original VM has been studied under several theoretical approaches, being the hydrodynamics formulation by Toner and Tu [5] one of the most influential. In this context we must also cite Bialek et al. [6] for their quantitative microscopic theory to describe starlings flocking based on experimental data.

Natural systems, such as the human heart rate [7], internet traffic [8], invasion percolation [9], financial time series [10–12], or space storms [13] are known to show phase transitions related to fractal properties usually measured by Hurst exponents. Recently, Zhao et al. [14] have studied the correlations present in a second order phase transition (2D Ising) and have found that the generalized Hurst exponent can serve as an early phase transition warning. In this work we study the temporal correlations present in the order parameter time series for the Vicsek model with vectorial noise undergoing a phase transition.

The remainder of this work is organized as follows: in Section 2 we detail the Vicsek model with vectorial noise; in Section 3 we discuss the DFA framework used to analyze the order parameter time series with results presented in Section 4. In Section 4.1 we discuss the presence and relevance of nonstationarities in the series at phase transition. Finally, the conclusions are detailed in Section 5.

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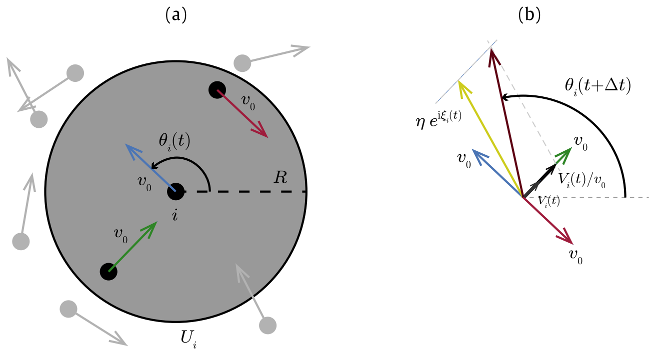

Fig. 1. The Vicsek model with vectorial noise. (a) Particles to be considered in the circular neighborhood of particle *i*. (b) Computation of V_i for particles in U_i and vectorial noise adding to find $\theta_i\,(t+\Delta t)$ $(v_0 = 1/2)$.

2. The Vicsek model with vectorial noise (VMVN)

We simulate N particles in a two dimensional box of sides L with periodic boundary conditions (a particle leaving the box from one side re-enters on the opposite side), moving at a fixed speed v_0 [1]. Each particle has a position $\vec{r}_i\,(t)$ and a uniform speed v_0. At $t = 0$ we [we distribute the N particles across the box by assigning random initial positions and angles (the angles θ_i are in the $[-\pi, \pi]$ range). Changes in particle velocity are due to angle $\theta_i\,(t)$. Now, for a given particle we center a circular neighborhood $U_i\left(\vec{r}_i; R; t\right)$ (where R is a maximal interaction distance for all particles) and calculate the average velocity among neighbor particles:

$$\vec{V}_i\,(t) = \frac{1}{k_i\,(t)} \sum_{j:\vec{r}_j \in U_i}^{k_i(t)} \vec{v}_j\,(t) \tag{1}$$

where $k_i\,(t)$ is the total number of particles in U_i (this includes the particle itself, as it is clarified in Fig. 1-a).

The spatial evolution of the particle will be affected by a noise factor; in this work we shall use the *vectorial noise* as proposed by Chaté et al. [15]. This is how a particle's angle is updated based on the average local velocity described in Eq. (1), and how this in turn updates velocity and position:

$$\begin{aligned} \vec{r}_i\,(t+\Delta t) &= \vec{r}_i\,(t) + \vec{v}_i\,(t)\ \Delta t \\ \theta_i\,(t+\Delta t) &= \text{Angle}\left\{\vec{V}_i\,(t)\,/v_0 + \eta\, e^{\mathbf{i}\xi_i(t)}\right\} \\ \vec{v}_i\,(t+\Delta t) &= v_0\, e^{\mathbf{i}\theta_i(t+\Delta t)} \end{aligned} \tag{2}$$

where the "Angle" function is defined as Angle $\left\{u_0 e^{\mathbf{i}\theta_0}\right\} = \theta_0$ (we here use the complex number representation of 2D vectors for simplicity), $\xi_i\,(t)$ is a random number uniformly distributed in the $[-\pi; \pi]$ range, η is the noise intensity and the time increment per simulation step is $\Delta t = 1$ (Fig. 1-b). Note that the updated positions $\vec{r}_i\,(t+\Delta t)$ are based on velocities prior to update $\vec{v}_i\,(t)$ (this is known as the *backward update rule* [16]).

2.1. The order parameter

The *order parameter* [1,17] (sometimes called *general consensus*) is an instantaneous value given by the set of agents' velocities according to

$$\phi_\eta\,(t) = \frac{1}{N v_0} \left\| \sum_{i=1}^{N} \vec{v}_i \right\|. \tag{3}$$

It should be noted that $\phi_\eta\,(t)$ is also a noisy time series. Simulated systems begin with a very low order parameter in initial conditions ($\phi_\eta\,(t=0) \sim 0$, disordered phase) since all particles start with velocity v_0 in random directions and after a sufficiently long simulation time τ, the system (away from coexistence noise) will reach a unimodal order parameter distribution. In order to establish if a proposed τ is appropriate, we study the curve $\left\langle \phi_\eta\,(t) \right\rangle$ (average across realizations). As follows,

1. Divide the interval $I = [\tau; T]$ in 10 sub-intervals (being T the total simulated time).
2. Calculate the set of slopes m of a linear regression of $\left\langle \phi_\eta\,(t) \right\rangle$ in each sub-interval.
3. Find the average slope $\langle m \rangle$.