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Insights into the macroscopic behavior of equity markets: Theory and application

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HIGHLIGHTS

- Introduce a macroscopic model of equity markets.
- Macroscopic sensors to detect abnormal activity.
- Valuable information is captured by macroscopic sensors during the flash crash day.

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ABSTRACT

In this research, we propose a macroscopic model of the equity market based on the physics of fluid dynamics. We develop sensors triggered by certain properties of the macroscopic variables, density and velocity, that can alert regulators to abnormal activity. Fluid flow in physics will be used to measure the irregularities found in the behavior of financial markets. Testing the proposed sensors on the day of a flash crash suggests further investigation in this area.

1. Introduction

Equity market crashes are defined as sharp declines in the value of market securities. The crashes are rare and hard to explain. In 1987, the US stock market dropped by about thirty percent over the course of four trading days. We may wonder what causes financial crashes and whether they signal market inefficiency. The research in this area is still growing, but fundamental aspects of crashes are poorly understood, including how to predict them [1].

As market exchanges have become more automated, with the adoption of limit-order systems, and as the capability of computing technology expands, high-frequency trading will become a relatively common practice. With the advancement of methods of execution, new abnormalities and irregularities have started to appear. These events raise questions about the stability and structure of financial markets [2]. In the last decade, stock markets and economies have experienced time periods that are characterized by sudden state transition and high unpredictability [3]

The flash crash is a poignant example of such irregularities. A flash crash is a mini-crash that is sudden and causes a sharp drop in the market within a short period of time [4]. The authors of [3] believe that flash crashes are not new phenomena, and that they in fact share similarities with previous crashes but exhibit different specifics. Given the aim of realizing profits by traders, one can expect more such crashes in the future [5]. An example of this is the recent flash crash of Amazon stock [6].

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The manuscript [7] argues that a flash crash is a failure in a large-scale complex socio-technical system. In 2007, the Large-Scale Complex Systems IT Systems Initiative was established to address issues in science and technology related to financial-market failures. The initiative constitutes the interactions of the following fields: complexity science, predictable software systems, high-integrity systems engineering, socio-technical systems engineering, organizational complexity, and novel computation approaches. This program, which resulted in some publications on system failures, was terminated in 2013, but the problem is still relevant. Although many papers have investigated the reasons for mini-crashes, only a few have provided a suitable model for predicting them [4,5].

The complex-systems approach, therefore, might provide insights for policy makers, academics, and market participants since the dynamic nature of complex systems is well suited to capture the nonlinear relationships between their inputs and outputs [3]. According to power-law statistics, small and large events belong to the same distribution, and it is believed that major catastrophes started out as small events that did not stop growing and became extreme events [8]. The scientific community considers extreme events to be unpredictable and in many cases hard to forecast. This view is emphasized by Bak in [9], where the concept of self-organized criticality was introduced. The notion blossomed when Taleb introduced the black swan concept in 2007 [10]. However, a recent study of financial crashes [8] suggests that markets undergo a phase transition before experiencing a crash. This hypothesis also suggests that equity markets exhibit signals referred to as dragon kings, which are extreme events that belong to the same class of certain other events but with an amplifying mechanism. In their article [11], Werner and his colleagues proposed examples where dragon kings can be detected. A recent method for detecting bubbles and crashes are reported in [12] where the authors show that a potential force is detected before a main crash in which it is considered as a "precursor" to a potential risk in the markets.

In this paper, we investigate the existence of market sensors, which monitor certain market measures and send out impulses in cases of abnormal activity. These measures are based on properties drawn from physics and modeled as a dynamical system to determine their future evolution. Physics and finance have interacted for a long time. The one-dimensional random walk was introduced in finance by Bachelier in 1900 after the venture of physicists into stock markets [13]. Bachelier's work was eventually forgotten, but it was reinvestigated by scientists in 1950 and later to describe various aspects of financial markets [14]. Their works have motivated significant research in options pricing theory, portfolio theory, and many other areas [14,15]. In 1900 Bachelier introduced the notion of Brownian motion in his Ph.D. dissertation, and Einstein used probabilistic models to explain the theory [16]. Brownian motion was first discovered by the biologist Robert Brown in 1827 [17] while studying microscopic pollen particles floating in water. Geometric Brownian motion has become the central mathematical model in pricing financial derivatives [16]. In the finance literature, a stock price $S(t)$ is assumed to be a stochastic process and follows a geometric Brownian motion given by

$$dS(t) = S(t)(\alpha dt + \sigma(t)dW(t)), \tag{1}$$

where α is the drift term, σ is the volatility of $S(t)$, and $W(t)$ is the standard Wiener process. The properties of a Wiener process can be found in any standard textbook on option pricing and financial mathematics.

In Eq. (1), we see an attempt to analyze a single random variable $S(t)$ in a dynamical system. This model is microscopic; it aims to model an individual stock and study its behavior. Even if this model represents the actual behavior of the underlying stock price $S(t)$, it does not capture the overall dynamics of the stock market. The purpose of our study is to define a macroscopic model with sensors that are triggered by abnormal market activity.

The increased interest in financial modeling has motivated a new line of research, in mesoscopic models [18], which in many cases consider a portfolio of stocks. Market indexes are a good example of mesoscopic models. Other mesoscopic models in statistical physics are used in modeling heterogeneous agents with a view to understanding their interactions. A well-known example of a mesoscopic model is a Fokker–Planck equation [18]. Several other methods and tools from statistical mechanics have been employed in financial markets [19–21]. They were implemented in the first place because they are capable of dealing with sophisticated environments where complex behavior arises from a rather simple interaction of some components. The purpose of this trend of research is to identify the universal and non-universal features of financial data [22]. A major area of research in the physics of finance is concerned with power tails, especially in wealth distribution, which was predicted by Pareto. The tools from statistical mechanics help to identify the features of tails in wealth distribution [23,24]. There have also been attempts to derive models with the tools of the kinetic theory of fluids [25,26]. Such models are based on partial differential equations (PDEs), which makes it possible to obtain general information and derive asymptotic behavior analytically [22]. An important application of fluid dynamics in the financial markets is introduced in [27]. There, the authors propose a new model for financial price movements in which the prices in the order book are described as colloidal Brownian particles.

Therefore, it is clear that the implementation of statistical mechanics and physics in finance is not new. Each method has its novelty and importance to the field. In both microscopic and mesoscopic models, the dynamical model, if it exists and is specified, is concerned with only a single stock or a portfolio of stocks. This approach might be useful if the aim is to study the stocks within the portfolio under study. An obvious drawback of such models is that even though the PDEs involved might provide a good prediction of the studied state behavior, it might be hard to verify with data. Also, those models might not capture the overall behavior of markets, as they consider a small subset of that behavior. Further, research based on PDEs with agents as the central argument are lacking in information, assuming that such information is not available to the public.

In this paper, we advance the literature by building a system of stochastic partial differential equations to analyze and monitor equity markets macroscopically. This model employs physics principles borrowed from fluid dynamics, and provides