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Impact of network randomness on multiple opinion dynamics

Vivian Dornelas [a], Marlon Ramos [b,*], Celia Anteneodo [a]

[a] Department of Physics, PUC-Rio, Rio de Janeiro, Brazil
[b] Gleb Wataghin Institute of Physics, Universidade Estadual de Campinas, São Paulo, Brazil

HIGHLIGHTS

- In-flow and out-flow multiple-opinion contagion dynamics are compared.
- Watts–Strogatz networks are considered to analyze the effect of shortcuts.
- Shortcuts favor the predominance of a winning opinion, in particular consensus.
- Undecided nodes can persist only in the in-flow dynamics.
- The number of undecided nodes can have a local maximum in the small-world region.

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ABSTRACT

People often face the challenge of choosing among different options with similar attractiveness. To study the distribution of preferences that emerge in such situations, a useful approach is to simulate opinion dynamics on top of complex networks, composed by nodes (individuals) and their connections (edges), where the state of each node can be one amongst several opinions including the undecided state. We analyze two different dynamics: the one proposed by Travieso and Fontoura (TF) and the plurality rule (PR), which are paradigmatic of outflow and inflow dynamics, respectively. We are specially interested in the impact of the network randomness on the final distribution of opinions. For that purpose, we consider Watts–Strogatz networks, which possess the small-world property, and where randomness is controlled by a probability p of adding random shortcuts to an initially regular network. Depending on the value of p, the average connectivity $\langle k \rangle$, and the initial conditions, the final distribution can be basically (i) consensus, (ii) coexistence of different options, or (iii) predominance of indecision. We find that, in both dynamics, the predominance of a winning opinion is favored by increasing the number of reconnections (shortcuts), promoting consensus. In contrast to the TF case, in the PR dynamics, a fraction of undecided nodes can persist in the final state. In such cases, a maximum number of undecided nodes occurs within the small-world range of p, due to ties in the decision group.

1. Introduction

Most opinion models proposed in the sociophysics literature [1–5] consider a binary variable, since diverse problems can be analyzed through the assumption of two single choices (e.g., for and against). However, in many everyday situations, we have to choose an option among several available ones with similar attractiveness, for example, choosing a movie, restaurant or buying a simple product in a supermarket. When we face such situations, without clear knowledge of the products

* Corresponding author.
E-mail addresses: vivian@aluno.puc-rio.br (V. Dornelas), marlon.ramos@pq.cnpq.br (M. Ramos), celia.fis@puc-rio.br (C. Anteneodo).

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offered, we tend to be influenced by friends, family and other contacts. There may be cases where each contact suggests a different product, and we remain undecided. Despite these are common situations, few studies of social dynamics address the possibility of multiple choices [6–11]. Therefore, there are still many open questions, one of them is about the effect of contact network topology and, particularly, its level of randomness. This scenario motivates the present work.

In a network, sites represent individuals and edges the possibility of interaction between the linked sites. To each site one attributes a state, that can evolve through the interaction with contact neighbors. As a prototypical network of connections between individuals, we use the network proposed by Watts and Strogatz (WS) [12] because it produces the small-world (SW) property that is observed in many real social networks. In this network, it is possible to adjust the level of randomness, characterized by a parameter p that controls random relinking of the connections in an initially regular lattice. It is known that the complex topology resulting from this rewiring can influence processes as those which are of interest in the present context [13–16].

We will consider that the changes of opinion are governed by rules appropriate to our problem of interest. Then, we consider variants of two paradigmatic rules of opinion dynamics, both contemplating the possibility of multiple choices, as well as the undecided state. One of the rules is a proposal by Travieso and Fontoura [17] (TF), where the "contagion" of preferences occurs from an individual towards his/her neighbors in the contact network (outflow dynamics). The other one is a plurality rule [11] (PR), where the transmission of preferences occurs in the opposite direction, from the neighborhood towards the individual (inflow dynamics). Their precise definitions will be given in Section 3. Moreover, for TF case, the update is done asynchronously, but for PR case, two forms of update, asynchronous and synchronous, are considered.

We will see that both rules can give rise to different final configurations, such as coexistence of many preferences, consensus, or yet, cases in which the quantity of undecided individuals is expressive. The final distribution of opinions in the population will be characterized basically by the f_w fraction of individuals who have adopted the alternative with more adepts and by the f_0 fraction of undecided individuals. These two quantities can be influenced by the randomness p of the network and by its average connectivity $\langle k \rangle$, or even, by the initial conditions. Therefore, we will vary these factors to show their impact on the final distribution of opinions.

The paper is organized as follows. The networks used and the dynamic rules are defined in Sections 2 and 3, respectively. The results of our analysis are presented in Section 4 and final remarks are done in Section 5.

2. Watts–Strogatz networks

To create a Watts–Strogatz (WS) network, we follow the standard procedure [12], starting from a regular ring of N nodes, each one with connectivity k, and using a rewiring probability p ($0 \leq p \leq 1$).

Two useful measures of a network structure are the agglomeration coefficient C and the average distance L, which are defined as

$$C = \frac{1}{N} \sum_{i=1}^{N} \frac{2m_i}{k_i(k_i - 1)}, \tag{1}$$

$$L = \frac{2}{N(N-1)} \sum_{i,j=1}^{N} d_{i,j}, \tag{2}$$

where k_i is the number of connections of node i, m_i is the number of connections between its nearest neighbors, and $d_{i,j}$ is the shortest distance between nodes i and j.

Depending on the value of p, the quantities C and L change, decaying to zero as p increases.

The SW property, characterized by high agglomeration and low average distance L, emerges for intermediate values of p, and can be defined as follows

$$\begin{aligned} p > p_1(\epsilon) &\longrightarrow L(p)/L_0 < \epsilon, \\ p < p_2(\epsilon) &\longrightarrow C(p)/C_0 > (1-\epsilon), \end{aligned} \tag{3}$$

where p_1 and p_2 are the values of p for which $L(p) = L_0\epsilon$ and $C(p) = C_0(1-\epsilon)$, respectively, for a given value $0 < \epsilon < 1$. Although there is no precise choice for ϵ, we set $\epsilon = 0.2$, to find the SW intervals that are highlighted in some of the figures of Section 4.

3. Opinion dynamics

The state of each i node, $1 \leq i \leq N$, is described by the variable S_i, that can take values $s_1, \ldots, s_q$, representing the q different opinions. It can also take the value s_0, when the individual does not adopt a defined option.

We start the dynamics with a network where all nodes have not a formed opinion yet, that is, $S_i = s_0$ for all nodes i. Then, we attribute opinions to randomly chosen nodes called "initiators". In order to make all alternatives equivalent, we consider the same number of initiators (I) for each opinion, hence, there is a total of qI initiators. Thereafter, opinions evolve