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Nonextensive triplets in cryptocurrency exchanges

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HIGHLIGHTS

- We discover nonextensive triplets in cryptocurrency exchanges.
- Cryptocurrency markets can be properly described by nonextensive statistics.
- This is the first formal link drawn between finance and nonextensive theory.

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ABSTRACT

Cryptocurrencies represent a new type of financial assets that are traded in a decentralized and transparent way. Recently, cryptocurrencies with large market capitalization (mostly Bitcoin) have been studied theoretically, but a deeper understanding of their underlying mechanisms remains elusive. Here we explore the nonextensivity of price changes for 20 cryptocurrency exchanges from 2013 until 2017. We discover nonextensive triplets in the cryptocurrency market, where the three associated values for Bitcoin are remarkably close to those of the logistic map near the edge of chaos. The current findings strongly indicate that the cryptocurrency market represents a system whose physics is properly described by nonextensive statistical mechanics. Our results shed light on the complex and volatile nature of cryptocurrencies, and establish the first formal link with the nonextensive theory.

1. Introduction

The remarkable growth of cryptocurrencies has drawn widespread interest from the media and investors alike in recent years. A cryptocurrency represents a decentralized digital cash system, or a new tradable asset, where there is no single overseeing authority. In contrast to traditional exchange rates, the system operates as an online computer network connecting all those who are involved, to process and check individual transactions [1]. Bitcoin emerges as the oldest and most important one, but many other cryptocurrencies have appeared since its inception, with the same underlying technology. While most cryptocurrency exchanges are clones of Bitcoin with different parameters, some (such as Ethereum and Ripple) consist of more significant innovations. The term 'cryptocurrency' refers to the fact that the system relies on cryptography to secure transactions and to create additional currency.

Diverse studies are being employed to address the complex nature of cryptocurrencies, such as descriptive statistics [2–4], fractals and multifractals [5–8], information theory [8,9], and network analysis [10,11]. Yet a complete understanding of the cryptocurrency market remains an open problem. The presence of large price volatilities [5,6], long-range temporal

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dependencies, and heavy-tailed distributions [7], indicate that cryptocurrencies deviate from the normal expectation into out-of-equilibrium states where the traditional statistical mechanics does not work. For a wide class of such systems, it has been shown that they can be described by nonextensive statistics [12] instead. The nonextensive theory thus forms a strong candidate for shedding new light on the cryptocurrency phenomena.

In this work we report evidence of nonextensive triplets [13] (that characterize a system that follows nonextensive statistics) in the cryptocurrency exchange market. This represents the first such discovery in finance. Price changes in cryptocurrencies are characterized by pronounced intermittency, meaning there is an irregular alteration of price dynamics. They also exhibit unusually long temporal dependencies, where the correlations decay slower than exponential. Lastly, tails in the probability distributions follow a power law with an upper limit of two for the exponent. Interestingly, we discover that the nonextensive triplet for Bitcoin is remarkably close to that of the logistic map near the edge of chaos. Our findings of these triplets in a vast number of cryptocurrencies strongly indicates that they fall within the realm of nonextensive statistical mechanics. These results suggest that perhaps an intuitively simple (albeit theoretically intractable) mechanism may be responsible for their widely volatile behavior, promising a deeper insight into the phenomenon.

The remainder of this paper is organized as follows: Section 2 reviews the nonextensive theory; Section 3 describes the cryptocurrency data; Section 4 presents the results and discussions; Section 5 draws the conclusions.

2. Nonextensive theory

The nonextensive theory represents a generalization of Boltzmann–Gibbs (BG) statistical mechanics in order to deal with nonequilibrium systems that were unaccessible before. Tsallis initiated the study of nonextensive statistical mechanics by proposing a nonextensive definition of entropy [14], that can satisfy Clausius' prescription [15] in situations where the standard BG entropy does not work. From a mathematical standpoint, nonextensive statistics can be described by its entropic formulation

$$S_q = -k\frac{1-\sum_i p_i^q}{1-q}, \tag{1}$$

which is obtained by substituting exponentials with q-exponentials

$$e_q(x) = [1+(1-q)x]^{1/(1-q)}, \tag{2}$$

and natural logarithms with q-logarithms

$$\ln_q(x) = \frac{x^{1-q}-1}{1-q}, \tag{3}$$

reducing to the BG entropy, the usual exponential and logarithm as $q \to 1$. The power law exponent q, also known as the *entropic number*, is intimately related to the microscopic dynamics and characterizes the degree of correlations in the system. The implications of this seemingly simple generalization to physical systems are enormous:

i Probability distribution functions (PDFs) acquire heavy tails that are proportional to q-exponentials, where the corresponding states are characterized by a parameter $q \equiv q_{\mathrm{stat}}$.
ii Stationary states exhibit less than exponential sensibility to initial conditions; small initial differences between neighboring states diverge in q-exponential form where $q \equiv q_{\mathrm{sens}}$.
iii Macroscopic variables decay slower than exponential to their equilibrium values, namely q-exponentially with $q \equiv q_{\mathrm{rel}}$.

Therefore, a stationary state that follows nonextensive statistics is characterized by a triplet of q-values $(q_{\mathrm{stat}}, q_{\mathrm{sens}}, q_{\mathrm{rel}}) \neq (1, 1, 1)$, or "$q$-triplet", that satisfies $q_{\mathrm{stat}} > 1$, $q_{\mathrm{sens}} < 1$, and $q_{\mathrm{rel}} > 1$ [13], or even the more rigid set of conditions $q_{\mathrm{sens}} \leq 1 \leq q_{\mathrm{stat}} \leq q_{\mathrm{rel}}$ [12]. A particularly important instance of nonextensive statistics is that of systems that find themselves out of thermal equilibrium but still form stationary states, which can be found in a great variety of complex systems. This triplet has been successfully explored in natural phenomena such as the ozone layer [16], solar plasma [17,18], El Niño/Southern Oscillation [19], geological faults [20], river discharge [21], and in artificial systems including the logistic [22] and standard [23] maps, and scale-free networks [24].

3. Data

The period under study goes from 2013, when there was more than one cryptocurrency in circulation, to late 2017. We analyze time series of the top 50 cryptocurrencies that appear in the website https://coinmarketcap.com/currencies/ by market capitalization as of November 2017. Since cryptocurrencies can last anywhere between a few months and many years, we select a subset of 20 cryptocurrency exchanges with sufficiently long times (>600 days). For each of the cryptocurrencies we calculate over the time interval of one day the logarithmic change in closing price $S(t)$,

$$R_t \equiv \ln\left(\frac{S(t+1)}{S(t)}\right) \tag{4}$$