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The consentaneous model of the financial markets exhibiting spurious nature of long-range memory

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HIGHLIGHTS

- The consentaneous model is tested with empirical bursting of volatility in Forex.
- Structure of the model suggests the spurious nature of the financial market memory.
- All model constituents are necessary to reproduce the observed bursting statistics.

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ABSTRACT

It is widely accepted that there is strong persistence in the volatility of financial time series. The origin of the observed persistence, or long-range memory, is still an open problem as the observed phenomenon could be a spurious effect. Earlier we have proposed the consentaneous model of the financial markets based on the non-linear stochastic differential equations. The consentaneous model successfully reproduces empirical probability and power spectral densities of volatility. This approach is qualitatively different from models built using fractional Brownian motion. In this contribution we investigate burst and inter-burst duration statistics of volatility in the financial markets employing the consentaneous model. Our analysis provides an evidence that empirical statistical properties of burst and inter-burst duration can be explained by non-linear stochastic differential equations driving the volatility in the financial markets. This serves as an strong argument that long-range memory in finance can have spurious nature.

1. Introduction

We have to acknowledge that current understanding of the financial fluctuations and the nature of microscopic market interactions remains limited and ambiguous [1–3]. This imposes a natural limits on estimating risk in the financial markets and is directly related to the complex market dynamics involved [4–6]. Statistical physics is a useful tool to deal with complexity in the financial markets [7–9] as a greater insight is achieved using advanced methods of empirical analysis [10–12,1].

A major problem in the modeling in finance is related to the double stochastic nature of the fluctuations in the real markets. First of all there is a wide consensus on the need to model the behavioral opinion dynamics of the traders in the financial markets [13–19] and many models proposed are able to explain the fat tails and the volatility clustering. Usually these models describe oversimplified stochastic behavior with a limited number of the parameters and statistically adjusted values, which are not universal for the various definitions of the financial variables and various other statistical

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properties. Seeking for the heuristic model with universal parameters it is necessary to combine endogenous (agent-based) fluctuations with exogenous noise arising from the information or the order flow. Starting from the phenomenological stochastic modeling of return [20] we have proposed the consentaneous agent-based and stochastic model [21] (further in the text we refer to this model as the consentaneous model), which reproduces probability density function (PDF) and power spectral density (PSD) of the absolute return in the financial markets. The endogenous dynamics of volatility in this model is based on the stochastic differential equations (SDEs) derived for the infinite number of agents with global herding interactions. The time series of the high-frequency return in this model are generated by combining endogenous volatility with exogenous Gaussian fluctuations. First of all the consentaneous model with the same set of parameters reproduces PDF and PSD of absolute return for various assets and different markets. The statistical properties of the consentaneous model scale in the same way as the empirical data does for different return time-scale.

Later it was shown that the consentaneous model is able to explain various statistical properties of the high volatility return intervals extensively studied before in [22–26]. Our empirical study [27] using the consentaneous model across a wide range of time-scales from one minute to one month has demonstrated that proposed concept of financial fluctuations allows to understand statistics of volatility return intervals. In that study it was shown that for the sufficiently high values of the threshold the PDF of volatility return intervals has universal scaling with the prevailing power-law exponent 3/2. This inspired us for the further empirical study of burst and inter-burst duration PDFs for the time series of trading activity and absolute return (see [28]), which are usually considered to have the long-range memory. The power-law with exponent 3/2 in burst and inter-burst duration PDF probably means that Markov processes might be behind the stochastic dynamics of financial markets.

Here we employ the consentaneous model to demonstrate how various noises overlap and coexist finally resulting in the observed statistical properties of the burst and inter-burst duration. Being based on a Markov processes the consentaneous model helps us to explain the spurious nature of the long-range memory in the financial markets. In Section 2 we shortly discuss the structure of the consentaneous model, in Section 3 we compare empirical and model statistical properties of the burst and inter-burst duration, in Section 4 we analyze the effect of the various noises included in the consentaneous model on the PDFs of the burst and inter-burst duration. Finally we conclude the results presented in this paper.

2. Consentaneous model of the financial markets

We have already used the consentaneous model [21] to reproduce and explain the statistical properties of the volatility return intervals [27] and to argue for the necessity of the exogenous noise in the modeling of financial markets [29]. Here we describe the model in a very general terms seeking to reveal its relevance to the problem of the long-range memory. As was demonstrated in [28,30] the burst and inter-burst duration PDFs help to discriminate between two different origins of the observed long-range memory. The fundamental power-law with exponent 3/2 indicates about one-dimensional Markov processes in the origin of fluctuations when deviations from this law might be related with true long-range memory processes such as fBm. Our preliminary empirical analysis of the FOREX data [28] confirmed the presence of power-law with exponent 3/2 for the time series of trading activity and absolute returns. Here we demonstrate how the consentaneous model can be used to show that the financial markets might be driven by the long-term stochastic process described by non-linear SDE.

First let us recall that time series of return $r_\delta(t) = \ln P(t) - \ln P(t-\delta)$ related with market price $P(t)$ in sufficiently short time period δ, of one minute order, is defined in the model as [21]

$$r_\delta(t) = \sigma(t)\omega(t), \tag{1}$$

here $\omega(t)$ denotes a Gaussian exogenous noise, related to the order flow fluctuations, and $\sigma(t)$ is the slowly varying endogenous volatility (assumed to be almost constant for the time windows of width δ). Volatility being result of agent dynamics itself is a double stochastic process defined by ratio $y(t) = \frac{1-n_f}{n_f}$ of chartists $1-n_f$ and fundamentalists n_f as well by the mood of speculative traders $\xi(t)$,

$$\sigma(t) = b_0(t)(1 + a_0|y(t)\xi(t)|), \tag{2}$$

here the empirical parameter a_0 determines the impact of the agent dynamics on the observed time series. We account for the daily seasonality observed in the real data by introducing a periodic time dependence [21] of volatility

$$b_0(t) = \exp[-(\{t \bmod 1\} - 0.5)^2/w^2] + 0.5, \tag{3}$$

where $w = 0.25$ quantifies the width of intra-day pattern. The most important part of this approach is related to the stochastic processes $n_f(t)$, $y(t)$ and $\xi(t)$, which can be modeled using ordinary SDEs and they are thus Markov processes. Nevertheless, even in this case Markov processes $\sigma(t)$ and $y(t)$ exhibit the long-range memory properties, such as power spectral density $S(f) \sim f^{-\beta}$ with $\beta \approx 1$. Let us recall the SDEs defining these stochastic processes in the consentaneous model

$$\mathrm{d}n_f = \frac{(1-n_f)\varepsilon_{cf} - n_f\varepsilon_{fc}}{\tau(n_f)}\mathrm{d}t + \sqrt{\frac{2n_f(1-n_f)}{\tau(n_f)}}\mathrm{d}W_f, \tag{4}$$