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Spectral analysis of time-dependent market-adjusted return correlation matrix

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HIGHLIGHTS

- We present an adjusted method for the eigenvalues of a time-dependent correlation matrix.
- We use the correlation matrix on an 18-year data set of 310 S&P 500-listed stocks.
- We obtain more stationary eigenvalue time series via market-adjusted return.
- Co-movement and polarization of financial variables are important.
- We find an approximate polarization domain, characterized by a smooth L-shaped strip.

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ABSTRACT

We present an adjusted method for calculating the eigenvalues of a time-dependent return correlation matrix in a moving window. First, we compare the normalized maximum eigenvalue time series of the market-adjusted return correlation matrix to that of the logarithmic return correlation matrix on an 18-year dataset of 310 S&P 500-listed stocks for small and large window or memory sizes. We observe that the resulting new eigenvalue time series is more stationary than the time series obtained without the adjustment. Second, we perform this analysis while sweeping the window size $\tau \in \{5, \ldots, 100\} \cup \{500\}$ in order to examine the dependence on the choice of window size. This approach demonstrates the multi-modality of the eigenvalue distributions. We find that the three dimensional distribution of the eigenvalue time series for our market-adjusted return is significantly more stationary than that produced by classic method. Finally, our model offers an approximate polarization domain characterized by a smooth L-shaped strip. The polarization with large amplitude is revealed, while there is persistence in agreement of individual stock returns' movement with market with small amplitude most of the time.

1. Introduction

It is important to analyze the relative behavior of N particles with respect to changes in physical quantities. Empirical correlation matrices appear in similar complex problems of stock markets. The correlation between stock returns has been a fundamental component to the mathematics of finance since Markowitz first introduced the theory of portfolio selection [1,2]. Since then, time-dependent correlation within and across markets has been considered in the dynamics of

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assets on various time scales ranging from minutes to years [3–17]. Recently, the methods developed in applied spectral analysis and random matrix theory (RMT) have received significant attention in the study of correlation in financial markets. Many of these studies have focused on evaluating theoretical predictions [18–22] and uncovering interesting substructures and behaviors in markets [23–25]. Generally, they decompose the correlation matrix into market, group (sector), and the Wishart random bulk (noise terms) where the largest eigenvalue corresponds to market. [26] examines the daily returns of 422 US stocks for the time period 1962–96 and analyzes the deviating eigenvectors corresponding to the eigenvalues outside the random matrix theory bounds after removing the effect of the largest eigenvalue. [26] argues that the second largest eigenvalue of the empirical correlation matrix corresponds to large capitalization stocks. [14] finds that the deviating eigenvectors are stable in time. Recently, [27] finds long-range magnitude cross-correlations in time series of price fluctuations for 1340 members of the New York Stock Exchange (NYSE) Composite using time-lag random matrix theory. [28] discusses noise spectrum of the correlation matrix eigenvalues and the role of non-stationarity using Brazilian assets.

A common method of analysis in these studies is to observe the time series of the maximum eigenvalue of the time-dependent return correlation matrix. Recent literature has begun to investigate the use of this maximum eigenvalue time series in risk management and trading strategies [29]. [29] successfully incorporates a two-directional maximum eigenvalue of the time-dependent correlation matrix of the logarithmic returns into a moment-based percentile classification algorithm as a measure of risk. This signal prevents the strategy from trading in time periods where the eigenvalue of current window is relatively either too low (silence around overvalued price level before a storm in market) or too high (panic in market). Moreover, [29] provides compelling evidence for the usage of 100-session windows for correlation analysis in real market applications. Although the Marchenko–Pastur formula [30] is not applicable for this window size, the resulting signal is informative because of the long memory related to quarterly earnings reports. In other words, random matrix theory may not be applicable directly for some empirical correlation matrices due to its independence assumption and limitation on window size. Pairwise correlations between assets matter [31]. The resulting strategy in [29] outperforms the proxy for market in out-of-sample Monte Carlo simulations with random subsets of assets on random subperiods of the dataset.

Despite these significant contributions in literature, some issues with the usage of the maximum eigenvalue of empirical return correlation matrices remain. One such issue is that the time series corresponding to the maximum eigenvalue is highly non-stationary. It is important to obtain a more stationary time series in mathematical or physical modeling. One of our main goals in this paper is to find a relatively more stationary time series of maximum eigenvalue by using appropriate transformations. We have not preferred global nonlinear over-transformations or under-transformations such as logarithmic transformation. Instead, we focus on local transformation using relativity. We compare the resulting two dimensional and three dimensional time dependent maximum eigenvalue distributions. We obtain a relatively more stationary time series in mean value and standard deviation by using time series of market adjusted return (MAR).

The remainder of this paper is organized as follows. In Section 2, we introduce market-adjusted return and formalize our modified method. In Section 3, we describe the dataset constructed in order to test this method. In Section 4, we present the results of experiments on this dataset that evaluate the effect of our modification on stationarity. We compare kurtosis, autocorrelation function, and three dimensional maximum eigenvalue distribution values for both methods. Section 5 concludes the paper.

2. Market-Adjusted Return (MAR)

One of the fundamental assumptions in time-dependent correlation analysis is that the covariance structure of the underlying multivariate return process may not be constant or even stationary [32]. Most studies thus far have focused on modeling the covariance structure of this return process directly. In portfolio management, however, the return process of a portfolio can be sometimes misleading. Even if an individual asset has a positive and increasing rate of return over some window, this does not necessarily make it more attractive than the market. Moreover, just because two assets have positive correlation of return does not imply that their excess returns relative to market are positively correlated. A market-adjusted return (MAR) of an asset is simply the difference between the asset's return and a market index's return. Risk-adjusted returns typically consider the amount of MAR per unit risk. We focus on the correlation between the daily returns of assets. Since there is no natural way to measure the risk of an asset from a single data point in an out-of-sample algorithm, we cannot normalize market-adjusted return by risk, for example standard deviation. However, we can still better capture the movements and deviation behavior of assets relative to the market by examining the correlation between the market-adjusted daily returns of assets. Market-adjusted returns and risk-adjusted returns are identical in sign, and thus the parity of dyadic asset relationships is still preserved.

Before describing our modified method, we present an overview of the original method as presented in previous literature. The method takes as input an $M \times N$ return matrix R, where $R_{i,j}$ is the logarithmic return of the jth asset on the ith period, and τ is the window or memory size for calculating time-dependent correlation. For each time $t \in (\tau+1, M)$, the windowed estimate of the mean $\hat{\mu}^{\tau}(t)$ and standard deviation $\hat{\sigma}^{\tau}(t)$ of each asset's return is calculated. We can explicitly