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Generalized AIC method based on higher-order moments and entropy of financial time series

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HIGHLIGHTS

- We modify AIC method by extending variance to high-order statistics.
- We use information entropy to promote generalize AIC method ulteriorly.
- A new perspective of analyzing the fluctuation of financial time series which is named AIC plane, is proposed in order to derive more accurate results.
- There are some interesting results of stock markets in different areas when using generalized AIC method.

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ABSTRACT

In this paper, a generalized method of traditional Akaike information criterion is proposed as a new measurement to compare and evaluate the volatility behaviors of time series in stock market. This new method is modified by extending variance to high-order statistics such as skewness, kurtosis, and permutation entropy, so as to demonstrate the different aspects of volatility behaviors of time series compared to low-order method. Furthermore, a new model of creating an AIC plane is proposed in order to derive more accurate results. Numerical simulations are conducted over synthetic data to provide comparative study. In addition, further reports about the results of distinguishing behaviors in stock market composite index of America, China and Hong Kong by using generalized AIC method and AIC plane are utilized to reflect the feasibility. Our results can effectively differentiate multiscale volatility details of time series from three areas.

1. Introduction

Sequences in complex systems, represented by financial markets, generally exhibit fluctuations on a wide range of time scales that depends on data [1–3], and its dynamical behaviors of the economic index are consistent with the underlying economic trends [4–13]. Various statistical techniques, models and theoretical methods have been introduced to characterize volatility in financial time series from different aspects, in order to reflect the features of the dynamics of financial markets [3,5,14–32].

The Akaike information criterion (AIC) is used primarily for estimating the quality of statistical model [33]. There is no doubt that increase the number of free parameters can improve the superiority of data fitting. While looking for models that can best interpret data but contain minimal free parameters, the AIC method encourages the benignity of data fitting but avoids Overfitting, which makes it better than other methods to some extent. Recently researches are trying to extend

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statistical models by replacing the variance to high-dimensional statistics so that the results will be more accurate because of the reduction of uncertainty during characterization process [34–40]. In addition, inspired by the AIC-Selector, which is a reliable tool for the automated detection of acoustic emission signal, researchers innovate to use AIC for the identification and assessment of defects in time-dependent series [41–43]. This method can be effective in selecting accurate models in the presents of outliers. In this article, we generalized the Akaike information criterion to high-order statistics such as skewness and kurtosis, so as to investigate the volatility details of financial time series in the case of higher order moments. Since entropy is a stability and robustness nonlinear measurement to characterize the complexity of a financial time series, and permutation entropy has been recently suggested as a novel measurement for nonlinear time series [44–49], we also try to modify AIC with permutation entropy. Moreover, inspired by the entropy plane [50,51], we creatively proposed a totally new method — AIC plane to demonstrate the volatility of financial time series with respect to nonlinear monotone transformations. By using logistic map and lozi map [52], we generate time series over synthetic and real data to simulate and inspect the consequence of our models.

The remainder of the paper is organized as follows. In Section 2, we describe the methodology of AIC, generalized AIC method, and AIC plane. Next, Section 3 presents the simulation of logistic map and lozi map. Section 4 outlines the database acquisition and preprocessing, and then presents the results of SAIC, KAIC and PEAIC that apply to real stock markets to analyze the volatility of time series. Finally, conclusion is demonstrated in Section 5.

2. Methodology

2.1. Generalized AIC method

AIC method has been used to estimating the quality of statistical model, since it optimizes a good trade-off between the complexity and the fitness of the model. In addition, it can be calculated directly from the signal itself [52]. According to the advantage shown above, this criterion has been used to determine the onset of seismic waves in seismology in combination with two autoregressive fits. On the basis of this assumption, an autoregressive-Akaike information criteria (AR-AIC) method has been used to detect P and/or S phases [53–55]. For the AR-AIC picker, the order of the AR process must be specified through trial and error, and the AR coefficients have to be calculated for both intervals. Compare to the AR-AIC picker, Maeda uses a different AIC picker [56], which can be calculated directly from the records itself without fitting them with the AR processes, and this direct AIC method defines an AIC value for each sample n of a signal:

$$AIC_n = n \cdot \ln\left(\sigma^2_{1,n}\right) + (N - n - 1) \cdot \ln\left(\sigma^2_{n+1,N}\right) \tag{1}$$

We utilize the method above to study financial time series of stock market. Moreover, in order to investigate the volatility scaling property of a complex dynamical system such as stock market in a new perspective, we extended the AIC method to higher moments and proposed new methods, namely SAIC method, KAIC method and PEAIC method.

The SAIC and KAIC procedure consists of four steps.

Step 1

Consider a stock time series $x = (x_1, x_2, \ldots, x_N)$, and we use a sliding variable n to divide it into two parts, which are denoted as $x_{1,n}$ and $x_{n+1,N}$ respectively.

Step 2

For each n ranging from 1 to N-1, we calculate the skewness and the kurtosis of two sequences $x_{1,n}$ and $x_{n+1,N}$ respectively.

Step 3

We standardize the data to ensure the resulting are valid in the AIC formula:

$A(i, j)$ is the skewness (or kurtosis) of all the data from x_i to x_j. In order to make the $A(i, j)$ standard to facilitate the processing, they are converted into:

$$\widetilde{A}(i,j) = \left\{ \frac{A(i,j) - minA(i,j)}{maxA(i,j) - minA(i,j)} + \frac{1}{N} \right\} \tag{2}$$

where N is the total number of our sample.

Step 4

By substituting what we got from step 2 into the generalized AIC equation as follows, we get $SAIC(n)$ and $KAIC(n)$ that take n as the independent variable:

$$SAIC\,(n) = n \times ln[\widetilde{A}_s\,(1, n)] + (N - n - 1) \times \ln[\widetilde{A}_s\,(n + 1, N)] \tag{3}$$

$$KAIC\,(n) = n \times ln[\widetilde{A}_k\,(1, n)] + (N - n - 1) \times \ln[\widetilde{A}_k\,(n + 1, N)] \tag{4}$$

where $\widetilde{A}_s\,(i, j)$ is the standard skewness, and $\widetilde{A}_k\,(i, j)$ is the standard kurtosis of all the data from x_i to x_j.

Besides, in view of the significant role of entropy when characterizing the complexity of a financial time series as a stability and robustness nonlinear measurement, we also try to modify AIC with permutation entropy, which is denoted as PEAIC.