



# A continuous and efficient fundamental price on the discrete order book grid

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## HIGHLIGHTS

- Model of liquidity provision with quote discretization and liquidity rebates.
- Empirical test of the model on 100 Nasdaq stocks.
- Estimator of the fundamental price based on the rebate adjusted volume imbalance.
- Our estimator outperforms other existing estimators.

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## ABSTRACT

This paper develops a model of liquidity provision in financial markets by adapting the Madhavan et al. (1997) price formation model to realistic order books with quote discretization and liquidity rebates. We postulate that liquidity providers observe a fundamental price which is continuous, efficient, and can assume values outside the interval spanned by the best quotes. We confirm the predictions of our price formation model with extensive empirical tests on large high-frequency datasets of 100 liquid Nasdaq stocks. Finally we use the model to propose an estimator of the fundamental price based on the rebate adjusted volume imbalance at the best quotes and we empirically show that it outperforms other simpler estimators.

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## 1. Introduction

We say that a market price is *efficient* if it unambiguously reflects all available information, is arbitrage-free, and unpredictable. But different types of microstructural frictions prevent the observed price to freely reflect the efficient price. Among these, the price discretization implemented in most modern markets plays a major role. As a result there is no reason to expect that either best quote, or the mid-price (their average), coincide with an efficient fundamental market price.<sup>1</sup> As a related aspect, it is worth noticing that, in contrast with lower-frequency, correlations of (mid-)price return at ultra-high frequency do not necessarily imply inefficiency, as it does not imply that a round-trip is profitable.

Efficient price dynamics are somewhat paradoxical because many other financial time series display long memory: A striking example is the auto-correlation of the transaction signs [1–4]. A central endeavour in market microstructure is

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<sup>1</sup> The term *fundamental price* is used in this paper, as well in a significant part of the high frequency econometrics literature, to indicate the price of the asset in absence of microstructure noise. Another equivalent terms are *fair price* and *efficient price*.

to reconcile the predictability of trade signs with price efficiency, i.e. to uncover the adequate price formation process. Following Glosten and Milgrom [5], Madhavan et al. [6] (henceforth MRR) developed a simple theory of price formation: Prices are impacted proportionally to the innovation in the transaction history (order flow). This alone removes all price predictability when transaction signs are correlated. Included in their theory is Glosten and Milgrom [5]'s assumption that market makers avoid ex-post regrets by setting a finite spread. Accordingly, the theory yields relationships between price impact, trade sign correlations, and the bid-ask spread.

Because MRR's model disregards the role of price discretization it is not a priori clear that MRR's predicted relationships between market variables hold also for large tick stocks.<sup>2</sup> Hitherto, only some of these predictions have been empirically assessed in the literature. We perform here the first systematic test of MRR's general relationships and explicitly study how the tick size affects their performance. Although MRR's model assumed quotes and prices to be continuous we find that some of the predicted relationships between market variables are surprisingly accurate also when the tick is large and discretization plays, a priori, an important role.

More specifically, in this paper we shall argue that MRR's original idea can be adapted to large tick stocks by postulating the existence of an underlying efficient continuous market price. From this assumption we derive price formation equations for large tick stocks which reproduce, on average, the classical MRR relations. Our framework also predicts that price discretization becomes in fact important for second order price statistics (i.e. covariances and correlations). Thus, our model of liquidity provision explains how relationships between quantities depending linearly on the price hold regardless of the price discretization, whereas the tick size heavily influences quantities depending quadratically on the price.

In our model, limit order queues deplete when the fundamental price moves outside a certain interval around the mid price. We find that the length of this characteristic interval is typically larger than one tick and the mid-price dynamics are characterized by an interesting "stickiness". We show that this behaviour can be entirely explained by the existing rebate structure offered by the exchange to liquidity providers.

Our framework provides a straightforward way to reconcile the assumption of an efficient fundamental price with the inherent properties of financial markets. Whereas many empirical studies have focused on market dynamics on ultra high frequency time scales, little attention has been paid to the spatial dimension of price dynamics in modern financial markets. Because most liquidity provision in financial markets is nowadays channelled through high frequency market makers, important aspects of the market quality are naturally related to the properties of price changes on the smallest temporal and spatial scales. Our work develops an important step towards a unified theory of price formation and liquidity provision in small and large tick order books. Our point of view differs significantly from the perspective taken in some of the previous literature on the subject, too. We believe that our empirical observations suggest that price changes are mostly governed by strategic considerations of liquidity providers who agree on a hidden fundamental price. For example, we observe that a queue depletion at the best price entails with a high probability a permanent price change. Order book models with purely stochastic dynamics, so called "zero intelligence models", are not capable of reproducing this simple fact, unless they benefit from additional assumptions.

In large tick stocks the volumes at the best quotes contain a significant amount of information about the direction of the market [7]. The second part of our paper hence proceeds by defining an approximate fundamental price in large tick stocks by using the *squares* of the available volumes at the best bid and ask. This proxy can take continuous values within and beyond the region spanned by the bid and ask. We argue that this quantity performs better, as a proxy of the fundamental price, than generalized linearly volume weighted prices previously discussed in the literature. We assess the performance of our proxy by showing that (i) it incorporates to a large extent the information conveyed by the state of the order book, (ii) it behaves very much as the fundamental price, in that it follows approximately the MRR price formation rule. Having this proxy at hand allows us to reach further than much of the previous price formation literature, in that we can filter out exogenous public information shocks from the fundamental price dynamics, and study their effects on the *future* order flow. An interesting empirical finding of the second part of this paper is that the sign of the next trade is positively correlated with the exogenous information shocks.

Tick sizes are not expected to decrease in the near future, since regulation authorities express their explicit desire to "prevent a race to the bottom" [8]. European regulation agencies currently aim to establish a pan-European tick size regime "to prevent the use of the tick size as a competition tool" [9] within the MiFID.<sup>3</sup> The introduction of MiFID II requires that standardized tick size bands, which depend on the valuation of the regulated security, must be adopted across Europe. In the United States, the SEC announced in a press release on May 6 2015 that it "approved a proposal [...] for a two-year pilot program that would widen the minimum quoting and trading increments...", i.e. the tick size, for stocks of some smaller companies [10]. This reflects a general mood inclined towards freezing the tick sizes or even increasing them. Such opinion of regulation agencies is supported by some academic research, although no clear consensus seems to have emerged [11]. To summarize, we expect that the problem studied in this paper will have a relatively high longeveity and continue to be a challenge to regulators and market participants over a reasonably long term future.

This paper is structured as follows: In Section 2 we discuss the relevant literature. Section 3 reviews the functioning of limit order books. Section 4 presents the empirical datasets used in this study. We analyse the empirical implications of the

<sup>2</sup> In a large tick stock the ratio between tick size and price is relatively large and the spread is almost always equal to one tick.

<sup>3</sup> The Markets in Financial Instruments Directive is the EU legislation regulating firms providing services to clients linked to 'financial instruments' (shares, bonds, units in collective investment schemes, and derivatives), and the venues where those instruments are traded.

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