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Stochastic dynamics and stable equilibrium of evolutionary optional public goods game in finite populations

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HIGHLIGHTS

- Stochastic stable equilibrium (SSE) of the optional Public goods game is studied.
- Evolutionary game dynamics is described as a multidimensional Markov process.
- Effects of parameters on the probability to choose different SSEs are investigated.
- Cooperation behavior in finite populations is revealed in the stochastic system.

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ABSTRACT

Continuous noise caused by mutation is widely present in evolutionary systems. Considering the noise effects and under the optional participation mechanism, a stochastic model for evolutionary public goods game in a finite size population is established. The evolutionary process of strategies in the population is described as a multidimensional ergodic and continuous time Markov process. The stochastic stable state of the system is analyzed by the limit distribution of the stochastic process. By numerical experiments, the influences of the fixed income coefficient for non-participants and the investment income coefficient of the public goods on the stochastic stable equilibrium of the system are analyzed. Through the numerical calculation results, we found that the optional participation mechanism can change the evolutionary dynamics and the equilibrium of the public goods game, and there is a range of parameters which can effectively promote the evolution of cooperation. Further, we obtain the accurate quantitative relationship between the parameters and the probabilities for the system to choose different stable equilibriums, which can be used to realize the control of cooperation.

1. Introduction

Public goods game, as a generalization of the two prisoners dilemma game, profoundly depicts the social dilemma problems that may exist in the interaction of multiplayer. That is, the conflict between individual rationality and collective rationality. As social dilemma problems are widespread in many fields of the real world [1], how to resolve the dilemma has been attracted extensive attention from the economics [2–6], psychology [7–11], evolutionary biology [12–15] and

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other disciplines of scholars. The focus of attention is to seek some appropriate cooperation mechanisms. In other words, through mechanisms to induce self-interested individuals cooperate spontaneously, so as to achieve the maximization of the collective interests. Evolutionary game theory, based on groups of individuals, introduces the thought of biological evolution into the strategies updating process. It describes evolutionary process of strategies in the population according to their payoffs, which provides an effective analytical framework for the evolution of cooperation.

At present, in the framework of evolutionary games, some cooperation mechanisms for the public goods game have been proposed, such as punishment mechanism [14,16–21], reward mechanism [4,22–26], the reputation mechanism [27–30], network interaction mechanism [15,31–36]. These mechanisms can facilitate the formation and evolution of cooperation to a certain extent. Some other factors on the cooperation have also been studied, such as group-size [37], selection of the distributional rules [38], onymous interaction [39], the co-evolution of strategy and ability [40], and so on. For a more detailed research progress on the human cooperation in evolutionary games, the reader can refer to the recent review article of Perc et al. [41].

This paper focuses on the optional participation mechanism, which was first proposed by Hauert et al. [42]. By introducing a kind of non-participation strategy in the game, individuals are free to choose whether or not to participate in the investment business of the public goods. The individual who chooses not to participate does not contribute to the investment nor obtain the cooperative benefit. It is found that the introduction of non-participants can resolve the cooperation dilemma in the public goods game to a certain extent [42]. In the case of allowing non-participants, the cooperation and the defection strategies can coexist. The optional participation mechanism has a strong background in reality. Because when some individual in the game observes that its opponents are unfriendly, it can choose not to participate in the game with its opponents. This mechanism has attracted the attention of many researchers since it was proposed. Hauert et al. [43] established a replicator dynamics model for the optional public goods game in infinite populations. They found that the three strategies of cooperation, defection and non-participation showed a cyclical evolution phenomenon similar to the dynamics of the "rock–paper–scissor" game. In the subsequent study, they established evolutionary public goods game models which contained both punishment and non-participation strategies based on replicator dynamics [44] and on the Moran process [21] respectively. In addition, in the case of allowing non-participants, other evolutionary dynamics or factors affecting the cooperation behavior of the population have also been extensively studied. For example, Song et al. [45] established an evolutionary model based on Logit dynamics in the public goods game. Rand et al. [46] studied the impact of anti-social punishment on the cooperation behavior in the public goods game. Valverde et al. [47] studied the influence of global oscillations on the spatial public goods game by introducing a simple kind of random mobility in a lattice sparsely occupied.

In evolutionary games, the most core concept is the evolutionary stable strategy (ESS) proposed by Taylor et al. [48]. ESS describes that if the majority of individuals in a group choose this strategy, then a small variation group consisting of other strategies cannot invade this group. From the system point of view, the ESS of the game is a stable equilibrium of the system. In an evolutionary system that based on replicator dynamics, the evolution of strategies is described by differential equations. The ESS can be determined by analyzing the stability of the differential equations at the equilibrium point. However, the replicator dynamics based on differential equations are applicable to the case of infinite size populations, and the concept of ESS is suit for the absence of continuous noise in the system. When the number of individuals is finite, the Moran process is often used to model the evolutionary process of strategies in the present research. Moran-based models use the fixation probability to analyze the evolutionary dynamics of only two types of strategies in the system, and it is generally necessary to assume weak selection in the analysis. For the evolutionary game model in finite populations, the readers can refer to literature [49–56].

In order to describe the continuous noise effect in the evolution of the system, the concept of stochastic stable equilibrium (SSE) was proposed, firstly by Young and Foster [57,58]. The difference between SSE and ESS can be described as follows. The ESS treats each stochastic perturbation as an isolated event, and only takes into account whether the system can return to the equilibrium after being subjected to a small disturbance. However, a system may continually be subjected to small perturbations which can affect the evolutionary process of games. Thus, the ESS of the system may not be stochastically stable. The SSE concept can better describe the stability of the equilibrium in a noisy environment. However, since the concept of the SSE has been proposed, there has been little literature on the analysis of the SSE till now. As far as we know, in the recent literature, Quan et al. [59,60] studied the SSE of the evolutionary games in a finite size population based on the Markov process. And other models basically based on the stochastic differential equations for analysis which assume an infinite size population, such as literature of Huang et al. [61] and Liang et al. [62].

In this paper, we analyze the SSE of the optional public goods game considering both of the continuous noise and the effect of finite size populations. In contrast to the differential equations and the Moran process based models, we use the stochastic evolutionary dynamics proposed by reference [63]. Different from the model in reference [63] that was applied to the 2×2 symmetrical games, we further develop it to the optional public goods game and generalize the Markov-based evolutionary process from one-dimension to multidimensional cases. The SSEs of the system under different combinations of parameters are analyzed by the limit distribution of the multidimensional Markov process. We demonstrate the influence of these parameters on the cooperation behavior of the stochastic system. Moreover, we obtained the accurate quantitative relationship between the parameters and the probabilities for the system to choose different SSEs which can be used to realize the control of cooperation.

The remainder of the paper is organized as follows. In Section 2, we introduce the stochastic evolutionary optional public goods game in finite populations, including the expected payoffs of each strategy in the game, the stochastic evolutionary