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Behaviour of traffic on a link with traffic light boundaries

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HIGHLIGHTS

- Traffic behaviours over time and space on links with time-dependent boundaries.
- Theory for short and medium time behaviour for links with signalised boundaries.
- Analytical results of the traffic flow in relation to system and signal parameters.
- Rapid signal switching results in some effective time-invariant boundaries.

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ABSTRACT

This paper considers a single link with traffic light boundary conditions at both ends, and investigates the traffic evolution over time with various signal and system configurations. A hydrodynamic model and a modified stochastic domain wall theory are proposed to describe the local density variation. The Nagel–Schreckenberg model (NaSch), an agent based stochastic model, is used as a benchmark. The hydrodynamic model provides good approximations over short time scales. The domain wall model is found to reproduce the time evolution of local densities, in good agreement with the NaSch simulations for both short and long time scales. A systematic investigation of the impact of network parameters, including system sizes, cycle lengths, phase splits and signal offsets, on traffic flows suggests that the stationary flow is dominated by the boundary with the smaller split. Nevertheless, the signal offset plays an important role in determining the flow. Analytical expressions of the flow in relation to those parameters are obtained for the deterministic domain wall model and match the deterministic NaSch simulations. The analytic results agree qualitatively with the general stochastic models. When the cycle is sufficiently short, the stationary state is governed by effective inflow and outflow rates, and the density profile is approximately linear and independent of time.

1. Introduction

Cellular Automata (CA) are a popular approach to study freeway traffic. Particularly widely used is the famous one-dimensional Nagel–Schreckenberg (NaSch) model [1]. This model may be considered on a ring or with reservoirs at the boundaries to model on- and off-ramps. A special case of the NaSch model is the asymmetric simple exclusion process (ASEP), in which car speeds are either 1 or 0.

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A significant challenge in extending the model to an urban traffic *network* is to incorporate intersections (nodes) with realistic traffic lights. It is well known that road network performance is strongly dependent on traffic signal parameters. The relation between them is complex and difficult to predict. Given that, we choose to study traffic behaviours on a link with traffic light boundaries, the most fundamental component of a road network.

In the sections below we will develop theory to describe a single link and implement traffic lights at the boundaries. Four important parameters determine the behaviour of traffic on the link; these are the cycle time, the split, the offset and the link length. The cycle time is the total time allocated to one green–red cycle of the traffic light, and the split determines how much green vs. red time is allocated within each cycle. The offset is the time difference between the cycle start times at the two boundaries.

Intuition suggests us that a short cycle time, or fast switching, will result in similar behaviour to that of a link without traffic lights but with effective in- and outflow rates proportional to the amount of green time at the boundaries. Large cycle times, compared to the relaxation time of the model, will lead to two pseudo-stationary regimes, one with a green light and one with a red light. The simulation results presented in this article confirm this intuition.

Cellular automata and density profiles

NaSch models with time-independent boundary conditions have been extensively studied, e.g., [2–4], and exact solutions have been obtained for the special case of ASEP corresponding to maximum speed $v_{\max} = 1$. With time-independent boundary conditions, there are three general macro-states: low density, high density and maximum flow. Which state the model attains depends on the inflow and outflow rates at boundaries.

Studies of systems with time-dependent boundaries have also been conducted using CA models, in particular NaSch, and mean field theories. The first such study appears to be [5], who considered a semi-finite ASEP (i.e., assumed as $(-\infty, 0]$) and time-dependent outflow rates and split time fixed at a half. There it was found that the density has a time-periodic stationary sawtooth structure. A similar problem was studied in [6]. Both [5] and [6] concluded that the most interesting behaviour, and most difficult to analyse, occurs when boundary parameters vary slowly, like realistic traffic lights with intermediate values of the cycle time. Woelki [7] considered ASEP with random sequential updates using a mean-field approximation. The author investigated density-dependent inflow rates, which models the scenarios of adaptive drivers and/or traffic control (such as ramp-metering). When the system density exceeds a threshold, the inflow rate reduces to a smaller value. As a result two additional phases may occur aside from the usual low/high density and maximum flow phases. When the inflow rate is in rapid alternation the controlled-density phase and co-existence phase come into existence.

Deterministic NaSch systems with a traffic-light-controlled out-boundary were considered by [8–10]. Compared with the sawtooth structure in [5], a rectangular density structure was found in [9], which was induced by the traffic light and was sustained due to the absence of stochasticity. In [8], the authors focused on the discussion of travel delays caused by traffic lights. They found a dependence between the road length and the travel time. Jia and Ma [10] studied a model similar to [8,9]. They derived a theoretical expression of the outflow if the red phase is longer than 1 time step. Otherwise, they found the system generated periodic orbits and the maximum flow could depend on the road length. Their study of the deterministic NaSch has a limited application, as the analysis was on the basis of assuming a saturated inflow, and is impossible to be generalised to the stochastic model. Tobita and Nagatani [11] used another deterministic CA model to study the impact of the cycle time, split and offset on the traffic flow through a series of traffic lights in a deterministic ring system with respect to the dynamic transition and the fundamental diagram (FD). Their model is less realistic than NaSch, due to the assumption of infinite acceleration.

Ito and Nishinari [12] studied the ASEP with parallel updates and a time-dependent outflow rate controlled by pedestrians crossing at an intersection. The pedestrian crossing is modelled as an $M/M/\infty$ queue with discrete time and thus the effective outflow rate becomes a function of the pedestrian arrival rate and exiting rate. They proposed two approximation methods for the study of traffic outflows and the phase diagram: an extended two-cluster approximation and isolated rarefaction wave approximation, which, respectively, produced results in good agreement with the simulations for sufficiently large and small pedestrian crossing speeds. When pedestrian signals are present, it was found that the average flow in the high density phase is approximately proportional to the effective green time over the cycle time. Other studies [13–15] analysed traffic through a sequence of traffic lights.

Variational theory and macroscopic fundamental diagrams

Daganzo and Geroliminis [16] studied the impact of traffic signal switching on road capacity using variational theory (VT) [17]. In particular, they found the relation between the road capacity and green signal fraction (i.e., green split or ratio) for short and long roads. Their work and the following studies mainly focused on applying VT and the capacity formula to study aggregated route/network performance, see [18–20], in particular the estimation of the macroscopic fundamental diagram (MFD). Other studies investigated the MFD and critical density in relation to various network and signal parameters including: cycle length [21–23], street length [21,24–27], signal phase split [21,25–27] and signal coordination [21,24,28].