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Description of Bose–Einstein condensate of cold gas in interaction through virtual states of non-condensate atomic components

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ABSTRACT

We study the nonlinear interaction between two species of Bose gases having a large mass imbalance. It is presented the situation, when at low temperature, one of gas component, with smallest mass, is Bose–Einstein condensed, while the other gas is not. We show that the new interaction Hamiltonian with temperature dependent potential part takes into account all binary exchange energy between the atoms of the smallest mass component through the virtual states of non-condensate components. The modification of the traditional phase transition representation of the number of atoms in the condensate as a function of the temperature is described by an anomaly in the low temperature branch of this dependence. This anomaly have the tendency of the increasing of the numbers of atoms in condensate with increasing of temperature for a small value of the relative parameter T/T_c, where T_c is the critical temperature of the phase transition.

1. Introduction

The effects of nonlinear interaction in dilute two-species Bose–Einstein gases have attracted the attention of many theoretical [1–8] and experimental [9–11] investigations. On the theory side [1–8], studies include density patterns, phase separation of the components, and the spectrum of collective excitations. The spatial separation of two immiscible quantum fluids in a trap has been analyzed in [1] and it is typified by a ball-and-shell structure in which one fluid forms a low density shell around the other and it depends on the relative strength of interspecies and single-species interactions. A general classification of possible spatial structures has been studied in Ref. [12]. On the experimental side, [9–11], the effect of mutual repulsive interaction in the dynamics of miscible Bose–Einstein condensate (BEC) of mixtures of two hyper-fine spin state of ^{87}Rb in magnetic traps has been explored in [9–11]. Studies of both miscible and immiscible two-component *Na* condensates were reported in Ref. [13]. Scissors-like collective oscillations in a dual-species BEC of ^{41}K and ^{87}Rb was reported in Ref. [14].

In this manuscript, we study the quantum system formed from two ultra-cold atomic subsystems in the presence of very large mass imbalance. We show that the interaction between particles in a single-species BEC changes, as a function of temperature, when the gas interacts with another atomic non-condensed subsystem with large mass imbalance. In this scenario, the non-condensed atoms play the role of a virtual subsystem that renormalizes the binary interaction between atoms in the BEC. We show that such interaction depends on the concentration of the non-condensate component and

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thus on temperature. The new temperature-dependent effective Hamiltonian of the system [15,16] predicts an increase of the order parameter for sufficiently small temperature. The control of interaction in a quantum system through the concentration of one of the component, it open new applications and studies of interacting quantum fluids [17]. These include disordered ultra-cold lattice gases, frustrated ultra-cold gases, spinor lattice gases, lattice gases in artificial magnetic fields, and, last but not least, quantum information processing in lattice gases.

2. Effective Hamiltonian obtained from the elimination of non-condensate atomic component

We consider a two-species Bose gases (that we denote as A and C) in free space with intra- and inter-species binary atomic interaction. The gas is described by a set of coupled equations for wave functions of each component. The Hamiltonian of this mixture of gases can be presented in the following form:

$$\begin{aligned}\hat{H} &= \int \frac{\hbar^2}{2m_c}\nabla\hat{\psi}_c^+(r)\nabla\hat{\psi}_c(r)dr + \frac{1}{2V}\int \hat{\psi}_c^+(r)\hat{\psi}_c^+(r')U_{cc}(r-r')\hat{\psi}_c(r)\hat{\psi}_c(r')drdr' \\ &+ \int \frac{\hbar^2}{2m_a}\nabla\hat{\psi}_a^+(r)\nabla\hat{\psi}_a(r)dr + \frac{1}{2V}\int \hat{\psi}_a^+(r)\hat{\psi}_a^+(r')U_{aa}(r-r')\hat{\psi}_a(r)\hat{\psi}_a(r')drdr' \\ &+ \frac{1}{2V}\int \hat{\psi}_a^+(r)\hat{\psi}_c^+(r')U_{ac}(r-r')\hat{\psi}_c(r)\hat{\psi}_a(r')drdr', \end{aligned} \tag{1}$$

where m_j is the mass of the jth component, V is the volume, and $U_{jk}(r-r')$ is binary interaction potential between the jth and kth components. We apply the Fourier transform of the Hamiltonian (1) and pass to the new form

$$\begin{aligned}\hat{H} &= \sum_p \frac{p^2}{2m_a}\hat{a}_p^\dagger\hat{a}_p + \frac{1}{2L^3}\sum_{p,p',q} U_a(q)\hat{a}_{p+q}^\dagger\hat{a}_{p'}^\dagger\hat{a}_{p'+q}\hat{a}_p \\ &+ \sum_p \frac{p^2}{2m_c}\hat{c}_p^\dagger\hat{c}_p + \frac{1}{2L^3}\sum_{p,p',q} U_c(q)\hat{c}_{p+q}^\dagger\hat{c}_{p'}^\dagger\hat{c}_{p'+q}\hat{c}_p \\ &+ \frac{1}{2L^3}\sum_{p,p',q} U_{ac}(q)\hat{c}_{p+q}^\dagger\hat{a}_{p'}^\dagger\hat{a}_{p'+q}\hat{c}_p. \end{aligned} \tag{2}$$

We have described the mixture of light and heavy Bose gases. According to the last experimental results [18,19] the condition of Feshbach resonance in which the energy of the bound state of an interatomic potential is equal to the kinetic energy of a colling pair of atoms may drastically change the critical temperature of the heavy atomic system. In this case, the critical temperature depends on the internal degrees of freedom of each atom, and the mass dependence of critical temperature of two subsystems in interaction (1) is shifted to the second study plan. However speaking about two subsystems, A and C in interaction (2), we may choose the situations for which the critical temperature of one subsystem, is smaller than the critical one in another system. In this situation, we can use the elimination method of operators of the system with smaller critical temperature (1)–(14) for such two subsystems in interaction.

Below we study the situation when the intra-species interaction between the subsystems C and A is larger than the inter-species interaction: $|U_{ac}(0)| \gg |U_{cc}(0)|, |U_{aa}(0)|$. Due to the mass imbalance, the light atoms from A subsystem have the bigger critical temperature, $T_c \sim \hbar^2 n^{2/3}/mk_B$, in comparison with the heavy atoms from C subsystem. As the condensate of A subsystem appears at more large temperature than in the C-one, we may eliminate the operators of non-condensate system C considering that the number of C atoms in the condensate of A subsystem is not so large and the C atoms leave quickly the condensate mode of A subsystem. This means that the C atoms remain dispersed in a large number of momentum in "p"-space (see exp. (12)) while the atomic subsystem A achieve the condensate mode. In such description, we can use Born–Markov approximation in the description of the interaction between the A particle through the C-one in a similar way as this procedure was used in the description of the effective interaction between the electrons through the phonon-modes in the Bardeen–Cooper–Schrieffer model of superconductivity [20]. Using the projection operator technique, we eliminate the operators of the non-condensate component. For this, we may formally divide the system Hamiltonian into two parts $\hat{H} = \hat{H}_0 + \hat{H}_I$, in which $\hat{H}_0$ contains the free Hamiltonian parts plus the interaction between each subsystem: A and A, C and C

$$\begin{aligned}\hat{H}_0 &= \sum_p \frac{p^2}{2m_a}\hat{a}_p^\dagger\hat{a}_p + \sum_p \frac{p^2}{2m_c}\hat{c}_p^\dagger\hat{c}_p + \frac{1}{2L^3}\sum_{p,p'} U_{ac}(0)\hat{a}_{p'}^\dagger\hat{a}_{p'}\hat{c}_p^\dagger\hat{c}_p. \\ &+ \frac{1}{2L^3}\sum_{p,p',q} U_a(q)\hat{a}_{p+q}^\dagger\hat{a}_{p'}^\dagger\hat{a}_{p'+q}\hat{a}_p + \frac{1}{2L^3}\sum_{p,p',q} U_c(q)\hat{c}_{p+q}^\dagger\hat{c}_{p'}^\dagger\hat{c}_{p'+q}\hat{c}_p. \end{aligned}$$

The interaction part of the Hamiltonian which described the interaction between subsystems C and A is described by the expression

$$\hat{H}_I = \frac{1}{2L^3}\sum_{p,p',q} U_{ac}(q)\hat{c}_{p+q}^\dagger\hat{a}_{p'}^\dagger\hat{a}_{p'+q}\hat{c}_p(1-\delta_{0,q}).$$