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Outer synchronization between uncertain networks with adaptive scaling function and different node numbers

Ling Lü, Chengren Li *, Liansong Chen, Suyuan Bai, Lianjun Ge, Liyu Gao, Changhui Han

School of Physics and Electronic Technology, Liaoning Normal University, Dalian 116029, China

HIGHLIGHTS

- The scaling function synchronization between uncertain networks is investigated.
- The drive and response networks may differ in the node numbers and the network topologies.
- The adaptive scaling function can be replaced by the adaptive law.
- The uncertain adjustment parameter and the uncertain coupling matrix element are identified.

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ABSTRACT

We investigate outer synchronization between uncertain networks with adaptive scaling function and different node numbers. In designed synchronization strategy, the drive and response networks may differ in the node numbers and the network topologies. Moreover, the adaptive scaling function can be replaced by the adaptive law. The uncertain adjustment parameter and the uncertain coupling matrix element are identified simultaneously in the process of synchronization. Finally, the numerical example is exploited to illustrate the effectiveness of the proposed synchronization criteria.

1. Introduction

In nature, many large-scale systems can be described as various complex networks. Therefore, the research on dynamical behaviors in various complex networks has become one of the hotspots in many science and technology fields. Especially, the synchronization problem of complex network has attracted more and more attention [1–6]. There have been many interesting research results on synchronization of complex networks. Some different kinds of synchronous types, for examples, complete synchronization [7–9], phase synchronization [10,11], projective synchronization [12,13] and cluster synchronization [14,15], have been discovered.

Most of the existing researches have focused on the synchronization within one network, which is regarded as inner synchronization. In contrast, the synchronization between two or among more complex networks, regardless of whether the inner synchronization is realized or not, is more ubiquitous in the practical applications and it is called as outer synchronization. Among them, some typical works related to outer synchronization of the networks have been reported. For instances, Anbuvithya et al. investigated the non-fragile synchronization for bidirectional associative memory neural networks with time-varying delays and derived sufficient conditions to guarantee the non-fragile synchronization [16].

* Corresponding author.
E-mail address: lnnulicr@aliyun.com (C. Li).

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In the Ref. [17], Ma et al. studied impulsive synchronization of fractional T-S fuzzy complex dynamical networks and a synchronization criterion is established by utilizing the comparison principle. Bagheri and Ozgoli investigated the impulsive projective and lag synchronization of uncertain complex networks with time-varying delays in both coupled and uncoupled terms [18]. Lü et al. achieved the outer synchronization between uncertain networks using Backstepping design, and the adaptive laws of uncertain parameters and the structure of control inputs upon nodes of the response network were determined according to stability theory [19].

It is worth noting that in many practical settings, it is so difficult to exactly know all network parameters and the network topology beforehand, therefore, it is very necessary to develop some effective strategies in order to identify the network topology and network parameter in the synchronization process. For that reason, many researchers have made great effort to address this problem and some valuable results have been obtained [20–23]. Especially, the existing research on outer synchronization of networks focused mainly on the situation that the node numbers in the response network is equal to that in the drive network. But in practical application, people often need to increase or decrease the node numbers at any time according to specific requirements. And the node numbers in drive and response networks are often unequal. Therefore, the research on outer synchronization between uncertain networks with different node numbers has stronger practicability. In addition, the scaling function synchronization between networks has attracted a great deal of attention. The so-called scaling function synchronization between networks means that the drive and response networks could be synchronized up to a scaling function, but not a constant. Obviously, the synchronization mechanism is safer in secure communication. Therefore, the research is of great value. At present, the focus of research is the scaling function synchronization between continuous networks, but the scaling function synchronization between discrete networks is rarely reported.

Motivated by the above considerations, we investigate outer synchronization between uncertain networks with adaptive scaling function and different node numbers. In designed synchronization strategy, the drive and response networks may differ in the node numbers and the network topologies. Moreover, the adaptive scaling function can be replaced by the adaptive law. The uncertain adjustment parameter and the uncertain coupling matrix element are identified simultaneously in the process of synchronization.

The rest of this paper is organized as follows: Section 2 gives a general description of proposed network models and some preliminaries. Section 3 presents a detailed proof for the synchronization criteria. In Section 4, a numerical example is exploited to illustrate the effectiveness of the proposed synchronization criteria. Finally, conclusions are summarized in Section 5.

2. Network models and preliminaries

Consider a discrete spatiotemporal network consisting of N_1 dynamical nodes and the state equation for each node is

$$x_i(m,n+1) = f(x_i(m,n)) + \sigma_i \sum_{j=1}^{N_1} a_{ij} x_j(m,n) \quad i = 1, 2 \ldots, N_1 \tag{1}$$

where $x_i(m,n) = (x_{i1}(m,n), x_{i2}(m,n), \ldots, x_{is}(m,n))^T \in R^s$ is the state vector of the node i and $f(x_i(m,n))$ is the dynamical function of the node i. σ_i refers to the coupling strength and $\sigma_i > 0$. $A = (a_{ij})_{N_1 \times N_1}$ denotes the coupling matrix representing the topological structure of the network, in which a_{ij} is defined as follows: If there is a link from node $i$to node $j(i \neq j)$, $a_{ij} = 1$; otherwise, $a_{ij} = 0$. The diagonal element of the matrix A is defined as $a_{ii} = -\sum_{j=1, j\neq i}^{N_1} a_{ij}$.

If taking the network given by the Eq. (1) as the drive network, the following network consisting of N_2 dynamical nodes is considered as the response network

$$y_i(m,n+1) = g(y_i(m,n)) + \sigma_i \sum_{j=1}^{N_2} b_{ij}(m,n) y_j(m,n) + u_i(m,n) \quad i = 1, 2 \cdots, N_2 \tag{2}$$

where $y_i(m,n) = (y_{i1}(m,n), y_{i2}(m,n), \ldots, y_{is}(m,n))^T \in R^s$ is the state vector of the node i and $g(y_i(m,n))$ is the dynamical function of the node i. $B = (b_{ij}(m,n))_{N_2 \times N_2}$ is the uncertain coupling matrix. $u_i(m,n)$ is the network controller. And we assume $N_1 \geq N_2$.

The synchronization error between the drive and response networks is defined as

$$e_i(m,n) = y_i(m,n) - h_i(m,n) x_i(m,n). \tag{3}$$

Among them, $h_i(m,n)$ is an adaptive scaling function.