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Social optimum for evening commute in a single-entry traffic corridor with no early departures

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HIGHLIGHTS

- Evening commute behaviors in traffic corridor are studied under SO principle.
- Properties of optimum inflow curve with general desired departure time distribution are deduced.
- Analytic solutions for common desired departure time in SO are obtained.
- Three numerical examples with different per unit cost of time are carried out.

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ABSTRACT

In this paper, we investigate the evening commute behaviors on the social optimum (SO) state in a single-entry traffic corridor with no early departures. Differing from the previous studies on evening commute, the dynamic properties of traffic flow are analyzed with the LWR (Lighthill–Whitham–Richards) model. The properties of optimum cumulative inflow curve with general desired departure time distribution curve are deduced, and then the analytic solutions for common desired departure time in SO are obtained. Three numerical examples are carried out to capture the characteristics of evening commuting behaviors under different values of time. The analytic and numerical results both indicate that the rarefaction wave originating from the first entry point influences the whole or part of the outflow curve. No shock wave exists through the commuting process. In addition, the cost curves show that the trip cost increases and the departure delay cost decreases with departure time, whereas the travel time cost first increases then decreases with departure time under the SO principle.

1. Introduction

To date, various models have been developed to study the evening commute problem. Roughly speaking, the studies can be divided into two groups, where some researches investigate the morning and evening commuting problems in isolation [1–5] and others focus on the day-long commuting problems [6–9]. In addition, most of the above studies are developed based on the classical bottleneck model [10].

However, the basic bottleneck model [10] treats the traffic congestion as a queue behind a single bottleneck with fixed capacity, so the dynamic properties of traffic stream and traffic jam cannot explicitly be investigated. Hence, the evolution of traffic flow in the evening commuting process cannot be reproduced in these studies. In fact, the propagation properties

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of traffic dynamics have prominent influences on commuters' trip cost, and then affect the departure time choices in the evening peak hours. What is more, many researchers [1–3,5] find that the morning and evening departure patterns of specific individuals were symmetric under the basic bottleneck model. However, it is still unknown whether this symmetry exists or not under the flow congestion model.

Therefore, in order to capture the dynamic properties of traffic flow in the evening peak hours, we in this paper investigate the commuting behaviors for evening commute in SO state with flow congestion model. As for the dynamics properties of traffic stream, researchers have proposed many models to explore the driving behaviors [11–18], traffic jam [19–24], and control strategies [18,25–28], but these models cannot reproduce the departure time choice behaviors in the evening commuting process under the SO principle. In fact, Newell [29] has developed the flow congestion model to describe the commuting behavior instead of the point bottleneck model and explicitly investigate the dynamic properties of congestion flow. As for the advantage in describing flow congestion, many extensions of this model have been developed [30–33]. Thus, our work is to follow this point of entry and further contribute improvement on evening commute. We will focus our attention on the optimum cumulative inflow curve under the SO principle. First, the properties of inflow rate are discussed for an arbitrary desired departure time distribution, then the analytical solutions for SO state with a common desired departure time are deduced, finally three numerical examples are carried out to further testify the analytical results.

2. Model

Fig. 1 presents a schematic diagram of evening commuting in a single-entry traffic corridor. We use the word "departure" to indicate a vehicle's departure from the work place (origin) and consequent entry into the corridor, and the word "arrival" to indicate a vehicle's arrival at residence (destination) and consequent exit from the corridor. The following settings are considered.

(1) Vehicles travel along a single-lane road with length l and constant width. The road has only one entry and one exit located at $x = 0$ and $x = l$, respectively.

(2) Point queue may develop at the entry point if the departure rate exceeds the road capacity. The distance between origin and entry point is ignored, so does the distance between exit point and destination.

(3) If we normalize the time such that the first departure occurs at time $t = 0$, then, t_f and $\overline{t'}$ (departure time and arrival time of the last vehicle) become endogenous.

(4) Trip cost of a vehicle is a linear function of travel time and departure delay time. It follows

$$\text{Trip cost} = \text{Travel time cost} + \text{Late departure cost}, \tag{1a}$$

$$C(t) = \alpha_1\tau(t) + \alpha_2\left(t - \bar{t}(t)\right), \tag{1b}$$

where $\tau(t)$ is the travel time of the vehicle departing at time t (which includes the time spent in a queue if it appears); α_1 is the per unit cost of travel time; α_2 is the per unit cost of late departure time; $\bar{t}(t)$ is the desired departure time of the vehicle departing at time t; $C(t)$ is the trip cost.

Next, we used the LWR (Lighthill–Whitham [34,35] and Richards [36]) model to study the SO solution for evening commute in a single-entry traffic corridor with no early departures, where the LWR model can be formulated as follows:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0, \tag{2}$$

where x denotes location; t denotes time; k, q are respectively the traffic density and flow. For solving the LWR model, an explicit relation between velocity and density is needed. For simplicity, we adopt the Greenshields' relation,

$$v = v_0\left(1 - \frac{k}{k_j}\right), \tag{3}$$

where v and k represent velocity and density respectively; v_0 is the free-flow velocity; k_j is the jam density. Considering the relationship among flow, density and velocity, i.e., $q = v \cdot k$, we have:

$$v = \frac{v_0}{2}\left(1 + \sqrt{1 - \frac{q}{q_m}}\right), \tag{4a}$$

$$k = \frac{k_j}{2}\left(1 - \sqrt{1 - \frac{q}{q_m}}\right), \tag{4b}$$

$$w = \frac{v_0}{q'(k)} = \frac{1}{\sqrt{1 - \frac{q}{q_m}}}, \tag{4c}$$

where $q_m = \frac{1}{4}v_0k_j$ is the capacity flow; w is the reciprocal of the wave velocity $q'(k) = \mathrm{d}q/\mathrm{d}k = v_0\left(1 - \frac{2k}{k_j}\right)$ normalized by free-flow velocity. Fig. 2 shows the velocity–density relation and the flow–density relation, where $k_m = 0.5k_j$. Note: in