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Relationships between centrality measures of networks with diameter 2

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HIGHLIGHTS

- There is a precise functional dependence of the closeness with the degree: it is a linear relation with slope equal to −1.
- A new intuitive approach of the concept of betweeness based of three contributions to this centrality.
- A relationship between the degree, the betweeness and the clustering coefficient.
- These new relations were verified in seven diameter 2 networks.

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ABSTRACT

From an intuitive analysis of the betweeness of networks in which we detail its components (not only the basic definition of the betweeness as the number of short paths through a node), we propose a relationship between the betweeness, the degree and the clustering coefficient of a node in networks with diameter 2. We verify the validity of this relationship on seven diameter 2 networks: the global migration network, the dual networks of the underground systems of Barcelona, Tokyo, London and three model networks. We show also that the dependence of the closeness with the degree is an exact linear relation with slope equal to −1.

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1. Introduction

The diameter of networks is a fundamental parameter of the efficiency of the network in transferring data, energy, goods or people etc. between the network nodes [1]. Therefore, the most efficient network should be of $d = 1$, but the amount of links require for such network is $[n(n-1)]/2$ which decrease the efficiency of the network. In scale free networks the diameter is proportional to ln ln N and may be considered ultrasmall world. Another type of network which is positioned between these two options is a network of diameter equal to 2. Networks of diameter 2 are ultrasmall worlds with only two degrees of separation. These networks are super-efficient in transferring people and goods around the world, and therefore it is not surprising to identify diameter 2 networks in airplanes networks and shipping networks [2–4]. Many of these companies uses a hub port or hub airport to connect all their flights or shipping for highly connectivity transfers. In subway, metro and mass-transport systems the topology of the network is design for transporting people in cities and metropolitan areas [5,6]. A good correlation was found between the design of the network (stations cover maximum land area with minimum links between them and minimum transfers) and the number of boarding's [7].

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We can also see a similar but slight different phenomena, where instead of hub station, there are hub lines which connects most of the stations and other (less connected) lines are feeding the main line.

These examples are of man-made networks which were designed for efficiency, but we can give another example of diameter 2 network which is self-organized. The global migration network is an example of a self-organized network which has similar characteristics of a tiny world and diameter 2 topology and properties [8].

Networks with diameter 2 have also been the subject of theoretical works. One can mention the work of Erdös et al. [9] on the maximum degree of such networks and the work of Bermond et al. [10] on densification of bus networks.

In this article, we propose a new relationship between threes main centralities: degree, betweeness and clustering coefficient.

2. The data

In this work we examine seven diameter 2 networks and we show that there are several common characteristics.

1. The world migration network [8]. Migration is one of the largest phenomena of humankind in the last decades, and the network created by this phenomenon is a global network which connects all the countries in the world and millions of people. In most of the global data bases, migrants are defined as people who do not live in their birth country. We used a database which was published by the World Bank for 2006 and 2010 (14, 20, 21) of bilateral migration. For our analysis, we performed reduction of the 2010 matrix from the 2006 matrix. That network reflects the flow of migrants during these four years (2006–2010) between every two countries that has migration link. In this network there is one country (Australia) which is connected to all the other countries resulting a diameter of 2. Another characteristic of this network is the division of the nodes between those with larger degree (more than 100) and those with small degree.

2. The two group's model. It mimics the precedent network in that there are two groups of nodes: those with small degree and those with large degree with values near the nodes number. These last groups form one hub. All the networks had a similar base of random links with similar density (35% of all possible links). The difference between the networks was the number of nodes of the large degree group which were connected to most (or all) the nodes.

3. The quasi full network—we did a similar analysis with an almost full network by randomly remove 5% of the links of a full network. This process created a quasi-full network of diameter 2.

4. We analyzed some dual metro networks [11]. A dual network is useful for the representation of complex transport networks [12]. It is the transformation of a direct network (were links connect nodes) to a network were a link which connects two nodes is define as a node and two nodes which are connected define as link.

Some subway networks have the very interesting property: in order to travel from station to another it is not necessary to use more than two transfer stations. For example a traveler begins his way from a station located on the line A , he transfers on the line B and after some stations in this line B, he transfers on the line C and continues on the line C where is his final station. He uses three lines which correspond in the dual network to three nodes and changes at two transfer stations which are in the dual network the two links between these three nodes. Since it is the largest path in the dual network, this means that the dual network has a diameter equal to 2. We looked for some subway systems and found that the dual metro networks of Barcelona, of Tokyo and of London have effectively a diameter equal to 2. We remark that it is not the case for other subway systems of other towns.

We emphasize that all the networks of this article enter in the category of small networks but they are very different from the small world networks already studied which belong to the category of scale free networks with a power law degree distribution. [1,13].

The largest network of the present work is the migration network such that the degree distribution can be easily determined. In [8] we showed that the degree distribution can be described by two Gaussian curves, one centered around 30 and the second around 130. The other network for which an histogram was determined is the quasi-full network. This histogram has approximately the shape of a Gaussian.

3. The signature of the diameter 2 network: the relation closeness versus degree

We consider the relationship between the degree and the closeness in (ultra) small networks with diameter 2. In this unique type of networks only two steps (or less) are needed to cross the network from side to side.

When one considers the variation of the closeness with the degrees, one finds the closeness, Co, decreases linearly with the degrees with a slope equal to -1. This is true for all the networks of this study. We verified by an independent measurement that effectively the diameter of all the networks is effectively equal to 2.

It is easy to understand this property if one recalls the definition of the closeness of a node. It is the sum of all the shortest paths between the node and all other nodes. A node has N_1 first neighbors and N_2 most remote neighbors such that the sum $N_1 + N_2$ is equal to N_t the total sum of nodes of the network. Since the diameter or the largest shortest path is 2, one has for

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