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The tug-of-war behavior of a Brownian particle in an asymmetric double optical trap with stochastic fluctuations[☆]

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HIGHLIGHTS

- A Brownian particle optically trapped in an asymmetric double potential surrounded by a thermal bath was simulated.
- The simulation results obtained at different temperatures indicate that the oscillation behavior of the particle can be treated as the result of a tug-of-war game played between the resultant deterministic force and the random force.

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ABSTRACT

A Brownian particle optically trapped in an asymmetric double potential surrounded by a thermal bath was simulated. Under the cooperative action of the resultant deterministic optical force and the stochastic fluctuations of the thermal bath, the confined particle undergoes Kramers transition, and oscillates between the two traps with a probability of trap occupancy that is asymmetrically distributed about the midpoint. The simulation results obtained at different temperatures indicate that the oscillation behavior of the particle can be treated as the result of a tug-of-war game played between the resultant deterministic force and the random force. We also employ a bistable model to explain the observed phenomena.

1. Introduction

In the early 1970s, Arthur Ashkin and associates reported their pioneering work [1,2], whereby, using only the force of radiation pressure from a focused laser beam, a single microsphere, whose size was approximately equal to the wavelength of the laser, was suspended against gravity and trapped in stable optical potential wells formed in the focal region of two perfectly aligned counter-propagating laser beams. In the 1980s, Ashkin and associates further investigated optical trapping manipulation [3–6], and demonstrated that a microsphere can be trapped in three dimensions in the waist of a highly focused laser beam. This phenomenon is now commonly referred to as optical tweezers. Since the pioneering work of Ashkin et al., laser-based trapping and manipulation of particles has been widely applied in the fields of biology, physics and chemistry [7–12]. Microscopic particles have continued to be the fundamental objects employed for theoretical and experimental studies of optical trapping. The uniform geometry of microspheres allows computational modeling of optical tweezers through generalized Lorenz–Mie theory [13,14]. Upon introduction into an optical trap, a particle will move into its equilibrium position and orientation. Modeling this equilibration process requires the calculation of the optical force and

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torque. The calculational complexity can be greatly reduced if we employ the T-matrix method, which requires a single calculation regardless of how the characteristic of the incident light may change [15].

Randomness is ubiquitous in the real world, and is widely observed in systems ranging from nanophase materials and biomolecules to social structures and financial markets, which can often be described according to stochastic processes [16–26]. Physical and biological system, due to the contemporary presence of random fluctuations and nonlinearity, often present counterintuitive phenomena, such as stochastic resonance [27,28], resonant activation [29–32], Noise Enhanced Stability [33]. To reveal the nature of these interesting phenomena, researchers have increasingly pursued a better utilization of random forces and system noise. The most fundamental model for studying stochastic systems is a Brownian particle suspended in a fluid. Here, a Brownian particle subjected to thermal agitation from collisions with surrounding fluid molecules provides a well-defined random background that is dependent on the temperature and the fluid viscosity [34]. However, a Brownian particle can also be subjected to deterministic perturbations by introducing optical forces, which makes it is possible for us to study the interplay between the deterministic and the stochastic contributions, and ultimately control the dynamics of small particles [7]. As such, optically trapped particles have become an important model in statistical physics for studying microscopic phenomena [35–37].

In the present work, we theoretically investigate the trajectory of a Brownian particle engaged in a tug-of-war behavior when confined in a double optical trap. Simulation results conducted under different temperatures are presented in Section 2. A bistable system is employed to illustrate this interesting phenomenon in Section 3. A brief discussion and conclusions are presented in Section 4.

2. The tug-of-war behavior of a Brownian particle

The motion of an optically trapped Brownian particle in one dimension can be described by the Langevin equation:

$$m\ddot{x}(t) = -\gamma\dot{x}(t) + kx(t) + \sqrt{2k_B T\gamma}\,\eta(t), \tag{1}$$

where $x(t)$ is the position of the particle at time t, m is its mass, γ is the friction coefficient, k is the trap stiffness, $\sqrt{2k_B T\gamma}\,\eta(t)$ is the fluctuating force due to random impulses from the many neighboring fluid molecules, k_B is Boltzmann's constant, and T is the absolute temperature. The noise $\eta(t)$ has the statistical property of a zero mean:

$$\langle d\eta(t)\rangle = 0. \tag{2}$$

The left side of Eq. (1) is the inertial term $m\ddot{x}(t)$, on the right side, $-\gamma\dot{x}(t)$ is the frictional term, $kx(t)$ is the restoring force, and $\sqrt{2k_B T\gamma}\,d\eta(t)$ represents the emerging white noise in a random environment.

We consider a spherical Brownian particle in water that simultaneously interacts with two optical traps, where the particle's trajectory would be affected by the randomly fluctuating force of the colliding water molecules and the deterministic optical force. In the simulation, we set $T = 293.36$ K, the water viscosity $\mu = 1 \times 10^{-3}$ Pa s, the radius of the particle $R = 0.8 \times 10^{-6}$ m, and the distance between the two traps is $d = 1.5R = 1.2 \times 10^{-6}$ m. We employ two trapping laser beams of different optical power outputs, $P_1 = 0.2 \times 10^{-3}$ W, $P_2 = 0.18 \times 10^{-3}$ W. Under these conditions, we simulate the motion of the Brownian particle in water with time step $\Delta t = 5 \times 10^{-3}$ s and the number of steps $N = 10^4$. Fig. 1 presents the two-dimensional trajectory of the particle, over the 50 s period of simulation, where blue lines represent the particle trajectory, red lines represent the incoming rays intersecting the particle, rose lines represent non-intersecting incoming rays, and orange lines represent the deflected outgoing rays. The figure indicates that the position of the particle is mainly localized within three regions near the positions $x = -0.6$ μm (at the laser beam of P_1), $x = 0$, and $x = 0.6$ μm (at the laser beam of P_2). The distribution of the particle along a line between the two laser beams is more clearly illustrated in Fig. 2, where we note that the particle resides below the line $x = 0$ frequently than above, indicating that the probability of finding the particle nearby of laser beam of power P_1, is greater than finding it nearby of laser beam of power P_2. We also note a fairly high probability of the particle residing directly between the laser beams at $x = 0$. Here, the laser powers at $x = -0.6$ μm and $x = 0.6$ μm attain a temporary balance over a short period, and the particle is not inclined to move in either direction. This behavior can be related to a tug of war game. The particle is subjected to uncertain collisions from the liquid on both sides, and the resulting random perturbation may cause the particle to jump from one region to the other, although the overall probability of residing at $x = -0.6$ μm is greater than at $x = -0.6$ μm.

We then decreased the simulation temperature to $T = 273.16$ K with $\mu = 1.792 \times 10^{-3}$ Pa s, while retaining equivalent values for all other parameters. The simulation results in this case are shown in Fig. 3, where, relative to the results obtained at $T = 293.36$ K, the particle trajectory is observed to be increasingly concentrated near the laser beam of power P_1 at $x = -0.6$ μm. This is also clarified by comparing Figs. 2 and 4, where the frequency of occupancy below the line $x = 0$ in Fig. 4 is even more significant than that shown in Fig. 2. In Fig. 4, the particle transits to the region nearby the laser beam of power P_2(i.e., at $x = 0.6$ μm) only in the time interval between 26ths and 33rds. Therefore, the probability of finding the particle near the laser beam of power P_1 is much greater than the laser beam of power P_2. This indicates that the deterministic force is dominant at this temperature.

We then increased the simulation temperature to $T = 313.16$ K with $\mu = 0.656 \times 10^{-3}$ Pa s, while retaining equivalent values for all other parameters. As shown in Fig. 5, the trajectory of the particle in this case tends to be balanced between the two laser beams, and the probability of transition to the region near the laser beam of power P_2 is markedly increased. Here, stochastic fluctuation is observed to counteract the directed transition, as represented in Fig. 6.