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Oscillating entropy and spin precession in the ensemble of qubits interacting with thermal systems

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HIGHLIGHTS

- The paper presents clear explanation of the oscillating entropy in a system of qubits.
- This explanation is based on the relationship between the entropy and the precession of the real or effective spins of the qubits.
- It provides new physical insight on the understanding of the von Neumann entropy in the systems of qubits.
- It may substantially advance interpretation of the theoretical results in the study of qubits interacting with their environment.
- The paper should be of interest to a broad readership of physicists as the notion of the von Neumann entropy is widely used in various fields of physics.

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ABSTRACT

We present a simple model which allows us to explain the physical nature of the oscillating entropy. We consider an ensemble of qubits interacting with thermal two-level systems. The entropy of the qubits oscillates between zero and the value of entropy of the thermal systems. We show that the oscillations of the entropy can be clearly explained by the precession of the real or effective spins of the qubits.

Interaction between the qubits and their environment is vital for quantum information processing [1]. One of the most important parameters describing an ensemble of qubits interacting with the thermal environment is the von Neumann entropy. In recent years the entropy of the qubits has attracted a lot of attention, especially, in connection with the qubit entanglement [2–5]. It was found that the qubit entropy can oscillate with time when the two qubits, initially in a superpositional state, interact with a single oscillator [6]. The entropy also oscillates when the two qubits, initially in the entangled state, interact with the two independent photon baths in isolated cavities [7]. In both cases the whole system (qubits plus a system interacting with the qubits) is in the pure quantum state.

While the phenomenon of the entropy oscillations have been described in [6,7] the physical nature of these oscillations remained obscure. In this work we suggest a simple model which clearly demonstrates the nature of the entropy oscillations. In our model a single qubit spin (Q-spin) interacts with a single thermal spin (T-spin). We will show that the qubit entropy oscillates with time between zero and the value of the entropy of the thermal system. The entropy of the thermal system does not change for the Ising interaction and remains, approximately, constant for the Heisenberg interaction. Thus, the entropy oscillations cannot be associated with the flow of entropy between the qubit and the thermal system. We will show that the entropy oscillations can be clearly explained by precession of the Q-spins.

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First, we consider the Ising interaction between the Q-spins and the T-spins. The Hamiltonian for the Q–T-spins can be written in terms of the Pauli operators

$$H = -E_1\sigma_{z1} - E_2\sigma_{z2} - J\sigma_{z1}\sigma_{z2}. \tag{1}$$

Here E_1 and E_2 are the Zeeman energies of the Q- and T-spins, respectively, and J is the interaction constant. We assume that initially the qubits are placed into the uniform superposition of stationary states so that the Q-spins point in the positive x-direction. The corresponding density matrix is,

$$\rho_1(0) = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{2}$$

The initial density matrix of the T-spins at temperature T is given by the expression: $\rho_2(0) = \mathrm{diag}(f_{00}, f_{11})$, where $f_{00} = Z^{-1}e^{-E_2T}, f_{11} = Z^{-1}e^{E_2/T}$, and Z is the partition function $Z = e^{-E_2T} + e^{E_2T}$. (Here and below we put $k_B = 1$ and $\hbar = 1$.) The 4×4 density matrix of the two systems is the tensor product: $\rho(0) = \rho_1(0) \otimes \rho_2(0)$. Solving the von Neumann equation,

$$i\dot{\rho}(t) = [H, \rho(t)], \tag{3}$$

we obtain the time dependent density matrix with the non-zero components,

$$\begin{aligned} \rho_{00} &= \rho_{22} = \frac{1}{2}f_{00}, \quad \rho_{02} = \frac{1}{2}f_{00}e^{2i(E_1+J)t}, \\ \rho_{11} &= \rho_{33} = \frac{1}{2}f_{11}, \quad \rho_{13} = \frac{1}{2}f_{11}e^{2i(E_1-J)t}. \end{aligned} \tag{4}$$

Tracing the density matrix over the thermal system we find the reduced density matrix of the quibit's ensemble:

$$\rho_1(t) = Tr_2\rho(t) = \frac{1}{2}\begin{pmatrix} 1 & a \\ a^* & 1 \end{pmatrix}, \tag{5}$$

$$a = f_{00}e^{2i(E_1+J)t} + f_{11}e^{2i(E_1-J)t}.$$

The reduced density matrix of the thermal ensemble obtained by tracing over the qubits does not change: $\rho_2(t) = \rho_2(0)$.

The von Neumann entropy of the qubits with the Ising interaction is given by the expression:

$$\begin{aligned} S_1(t) &= -Tr\{\rho_1(t)\ln(\rho_1(t))\} \\ &= \frac{1}{2}X\ln\left(\frac{1-X}{1+X}\right) - \frac{1}{2}\ln\left(f_{00}f_{11}\sin^2(2Jt)\right), \end{aligned} \tag{6}$$

where,

$$X = (1 - 4f_{00}f_{11}\sin^2(2Jt))^{1/2}. \tag{7}$$

We will assume the following relation between the parameters of the Hamiltonian: $E_1 \ll J \ll E_2$. In this case the entropy (6) oscillates between zero and the value of the entropy of the thermal system. (See Fig. 1.) On the other hand, the entropy of the thermal system S_2 does not change with time:

$$\begin{aligned} S_2 &= -Tr\{\rho_2\ln\rho_2\} \\ &= -(f_{00}\ln f_{00} + f_{11}\ln f_{11}). \end{aligned} \tag{8}$$

In order to explain the entropy oscillations we will consider the precession of the Q-spins. The transversal component of the Q-spins computed with the reduced density matrix (5) is exactly the same as that computed with the full density matrix (4):

$$\begin{aligned} \langle\sigma_+\rangle &= \langle\sigma_x + i\sigma_y\rangle = Tr\{\sigma_+\rho_1(t)\} \\ &= f_{00}e^{-2i(E_1+J)t} + f_{11}e^{-2i(E_1-J)t}. \end{aligned} \tag{9}$$

The graph of the precession amplitude $|\langle\sigma_+(t)\rangle|$ is shown in Fig. 2 for the same values of parameters as in Fig. 1. One can see that the maxima of the precession amplitude correspond to the zero entropy and the minima correspond to the maximum entropy.

The physical reason of this correspondence is the following. Assume, for simplicity, that $E_1 = 0$. Then, as one can see from Eq. (5), the precession frequency of all Q-spins in the ensemble is the same and equals $2J$. The precession is clockwise when the T-spin, interacting with the Q-spin, is in its ground state and counterclockwise in the opposite case. At zero temperature all the Q-spins precess clockwise, i.e. the precession is circular polarized. At any instant of time all the Q-spins point in the