Physica A 502 (2018) 195–200

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Robustness of networks with assortative dependence groups

Hui Wang [a,b], Ming Li [c], Lin Deng [a,b,*], Bing-Hong Wang [d]

[a] School of Computer and Information, Hefei University of Technology, Hefei, 230009, PR China
[b] Information construction and development center, Hefei University of Technology, Hefei, 230009, PR China
[c] School of Engineering Science, University of Science and Technology of China, Hefei, 230026, PR China
[d] Department of Modern Physics, University of Science and Technology of China, Hefei, 230026, PR China

HIGHLIGHTS

- A percolation model on networks with assortative dependence group is proposed.
- The assortativity makes the nodes with large degrees easier to survive.
- Exact solutions to the critical point in agreement with the simulation results well.
- The critical point has an abrupt change when $g < 10$.

ARTICLE INFO

Article history:
Received 12 September 2017
Received in revised form 8 December 2017

Keywords:
Cascading failures
Assortativity
Percolation
Dependence group

ABSTRACT

Assortativity is one of the important characteristics in real networks. To study the effects of this characteristic on the robustness of networks, we propose a percolation model on networks with assortative dependence group. The assortativity in this model means that the nodes with the same or similar degrees form dependence groups, for which one node fails, other nodes in the same group are very likely to fail. We find that the assortativity makes the nodes with large degrees easier to survive from the cascading failure. In this way, such networks are more robust than that with random dependence group, which also proves the assortative network is robust in another perspective. Furthermore, we also present exact solutions to the size of the giant component and the critical point, which are in agreement with the simulation results well.

1. Introduction

Many real systems are presented in the form of networks, including social networks, computer networks, metabolic systems, food webs and traffic systems [1–3]. In the last decades, the robustness of these networked systems becomes one of the important topics in networks science, for which the percolation model is one of the most commonly used models [4–6]. In theory, the network percolation model describes the behaviors of the connected clusters of a network after a fraction $1-p$ of nodes are removed. This model usually exhibits a second order phase transition, that is the size of the giant component continuously decreases to zero while p decreases to the critical point p_c. The critical point p_c is usually used to evaluate the robustness of the network. The larger the critical point p_c is, the more fragile the network is, and vice versa. For some networks (such as random networks), the critical point can be solved exactly [7,8].

Recently, some researches focus on the cascading failures induced by the dependence between nodes, and the concept of dependence group (link) has been proposed [9,10]. The dependence group is a set of nodes, in which if one of the nodes

* Corresponding author at: School of Computer and Information, Hefei University of Technology, Hefei, 230009, PR China.
E-mail addresses: wanghui@hfut.edu.cn (H. Wang), minglichn@ustc.edu.cn (M. Li), denglin@hfut.edu.cn (L. Deng), bhwang@ustc.edu.cn (B.-H. Wang).

https://doi.org/10.1016/j.physa.2018.02.150

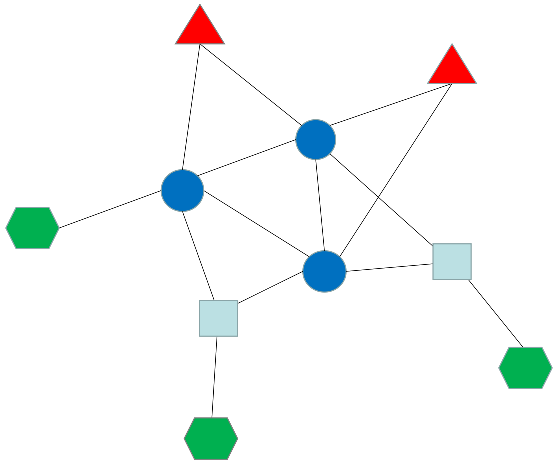

Fig. 1. Networks with assortative dependence groups. The network is composed of nodes and connectivity links, and each node belongs to a dependence group. Nodes in the same group have the same degree. We use different color and sharp to label different groups.

fails, the group will fail, i.e., all the other nodes in the group fail. Bashan et al. find that the robustness of such networks is determined by the size g of the dependence group [10,11]. A larger g could make such networks much more fragile than the one without dependence group ($g = 1$). This is not hard to understand, since the dependence group amplifies the destructive power of the cascading failures. More interesting, instead of the second order percolation transition, the first order percolation transition can be found for $g > 1$.

After that, a lot of researches are going into why real networks with dependence are not so fragile as the model, or how to protect such network system. Li et al. show that the overlapping of the connectivity and dependence links ($g = 2$) and the asymmetric dependence can make the networks with dependence groups more robustness [12,13]. Liu et al. study the percolation in coupled networks with both inner-dependence and inter-dependence links, they find that when the numbers of the two types of dependency links differ significantly, the coupled networks system is fragile and makes a discontinuous percolation transition [14]. Bai et al. study the robustness and vulnerability of networks with dynamical dependence groups, they find that an abrupt percolation transition exists, corresponding to the dynamical dependence groups for a wide range of topologies [15]. Kong et al. find that the heterogeneous dependence strength makes the system more robust, and for various distributions of dependency strengths both continuous and discontinuous transitions can be found [16]. In our previous work [17], we also point out that when the dependence group is not sensitive to the failures of a small fraction of its members, the size of the dependence group is no longer the determining factor of the network robustness, and a network with larger dependence groups could also be robust. In addition, this type of cascading failures is also studied in the form of interdependent networks and multi-layer networks, which also shows the fragile of networks when nodes depend on each other [18,19].

In real networks, the assortativity has already been proved as one of the important characteristics [20,21]. Taking this into consideration, we think that the nodes in one dependence group may also have the same or similar degree. In this paper, we will study the cascading failures of networks with assortative dependence groups. Without loss of generality, nodes are randomly divided into dependence groups with size g by their degrees as shown in Fig. 1. The nodes in the same group have the same degree. In a dependence group, when more than a fraction β of nodes fail, the group will fail, i.e., all the other nodes in the group fail.

The paper is organized as follows. In the next section, we will give the analytical results of our model. And then, we perform simulations on Erdős–Rényi (ER) network and scale-free (SF) network. The discussion and conclusion are given in the last section.

2. Analysis

The iterative process begins by randomly removing a fraction $1 - p$ of nodes of the network as the initial failed nodes. Then, links connecting with these failed nodes will also fail, which could lead to the other nodes disconnect to the network. This process is called percolation step. At the same time, if the fraction of the failed nodes in a dependence group is larger than β, the group will fail, i.e., all the other nodes in this group fail. This process is called dependence step. Once the cascade process is triggered, the two steps will occur alternately until there is no further splitting of nodes.

We use parameter R to represent the probability that a randomly chosen link connects to the giant component. Then, a node with degree k will connect to the giant component with probability $1-(1-R)^k$. Averaging over the degree distribution,