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Linear stability and nonlinear analyses of traffic waves for the general nonlinear car-following model with multi-time delays

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HIGHLIGHTS

- The general nonlinear car-following model with multi-time delays is proposed to describe the reactions of vehicle to driving behavior.
- Platoon stability and string stability are introduced and stability criteria are obtained.
- Burgers equation and KdV equation and their solutions are derived by adopting the reductive perturbation method.
- The properties of typical optimal velocity model with time delays are investigated using both analytic and numerical methods.

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ABSTRACT

In this paper, the general nonlinear car-following model with multi-time delays is investigated in order to describe the reactions of vehicle to driving behavior. Platoon stability and string stability criteria are obtained for the general nonlinear car-following model. Burgers equation and Korteweg de Vries (KdV) equation and their solitary wave solutions are derived adopting the reductive perturbation method. We investigate the properties of typical optimal velocity model using both analytic and numerical methods, which estimates the impact of delays about the evolution of traffic congestion. The numerical results show that time delays in sensing relative movement is more sensitive to the stability of traffic flow than time delays in sensing host motion.

1. Introduction

Time delays impair the driving performance and stability whether human driving or adaptive cruise control (ACC), which is a challenging topic in traffic science and engineering [1]. These delays affect the stability of traffic flow and ultimately impact safety and economy [2]. This theme has been concerned broadly, which is crucial for the influence of mechanism and controller design on the traffic flow [3].

Instabilities of traffic flow will result in traffic waves because of time delays in the actual traffic [4,5]. It is important to reveal that there exist various complex instability mechanisms in traffic flow due to delays [6,7]. Since 1950, delays in time-continuous models have been studied widely. Car-following models have drawn great attentions since that depicts

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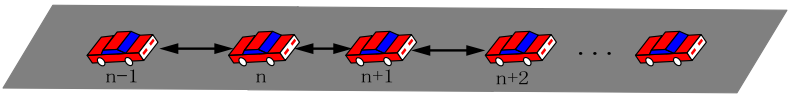

Fig. 1. The platoon of vehicles.

the dynamics of individual vehicle from the microscopic perspective [8]. Optimal velocity (OV) model with time delays was proposed by Bando et al. [9] and interpreted many sophisticated dynamical phenomena, e.g., traffic waves, clusters. Ref. [10] investigated microscopic traffic model considering driver's reaction time and found oscillations of velocity induced by slower vehicles. The local and global bifurcations of car-following model with delays are investigated. Different periodic bifurcation and Hopf bifurcation are analyzed by Orosz et al. [11,12]. Ref. [1] systematically investigated delays, inaccuracies and anticipation of driving behavior in microscopic traffic models. Two delays causing instability of traffic flow were revealed in Ref. [13]. Ref. [14] took two different time delays in sensing headway and velocity into account. Ref. [15] investigated the effect of reaction-time delay of drivers upon traffic flow. Ref. [16] considered delay of the driver's response in sensing headway to describe the traffic jam and presented the connection between the time-dependent Ginzburg–Landau (TDGL) equation and the modified Korteweg de Vries (mKdV) equation. Those contributions investigate that delays affect the evolution of traffic flow from different perspectives.

The methods of linear stability analysis are provided considering that a small perturbation of steady state will propagate with time [17–20]. For larger perturbations, nonlinear effect of traffic flow needs to be used to characterize wave profiles. The nonlinear equations of traffic flow could derive the nonlinear characteristics [21–24] such as solitary waves, kink waves and triangular waves. Based on microscopic traffic models and its extended models, many researchers investigated various properties of the traffic flow [25–32].

A large number of works proved that time delays play an important role on the property of the traffic flow, especially the stability. We are interested in stability of the general nonlinear car-following model with multiple delays. In this paper, the general nonlinear car-following model with multi-time delays is proposed to describe the reactions of vehicle to driving behavior. Platoon stability and string stability are introduced and stability criteria are obtained. Burgers equation and KdV equation and their solutions are derived. The properties of typical optimal velocity model with time delays are investigated using both analytic and numerical methods, which estimate the impact of delays about the evolution of traffic congestion.

The remainder of this paper is structured as follows. The general nonlinear car-following model with multi-time delays is introduced in Section 2. Section 3 analyzes the linear stability and obtains platoon stability criteria and string stability criteria. Burgers equation and KdV equation and its solutions are demonstrably formulated in Section 4. In Section 5, full velocity difference model with multi-time delays is seen as case study. The evolutions of traffic flow are investigated by using analytical and numerical methods. In Section 6, we conclude the results of this work.

2. Mathematical model

Many scholars have investigated some interesting and meaningful car-following models and the relevant works which focus on driving behavior of vehicle group moving on a single road without overtaking and lane changing shown in Fig. 1. Mathematical model of vehicle group can be expressed as nonlinear dynamical systems:

$$\frac{dv_n(t)}{dt} = f(v_n(t-\tau_1), \Delta x_n(t-\tau_2), \Delta v_n(t-\tau_3)). \tag{1}$$

where x_n is the position of the nth vehicle, v_n is the velocity of the nth vehicle. Headway distance $\Delta x_n = x_{n+1} - x_n$, velocity difference $\Delta v_n = v_{n+1} - v_n$. The acceleration has to be given as the nonlinear function of stimuli that are usually the headway distance Δx_n, the velocity difference Δv_n and the vehicle's own velocity v_n. Where τ_1, τ_2, τ_3 represent reaction delays to different stimuli.

Eq. (1) thus mimics how each vehicle accelerates or brakes in response to its own velocity and the relative motion of the preceding vehicle. To make the model more tractable, simple relations may be assumed between the different delays. Due to different levels of perception, delay of vehicle velocity is differ from velocity difference and headway obtained by driver or sensors [33,34]. So, we set two delays: τ_1 and $\tau_2 = \tau_3$.

The velocities and the headways are time-independent in equilibrium state, which satisfies:

$$\Delta x_n(t) = h_n^*, \Delta v_n = 0, v_n(t) = v_n^*. \tag{2}$$

We require

$$f(v_n^*, h_n^*, 0) = 0 \text{ for all } h_n^* > 0. \tag{3}$$

where v_n^* and h_n^* are the speed and headway in equilibrium state, respectively.

It follows that there is steady driving solution as:

$$x_n(t) = x_0 + nh_n^* + tv_n^*. \tag{4}$$