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A riddled basin escaping crisis and the universality in an integrate-and-fire circuit

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HIGHLIGHTS

- A crisis was observed for a discontinuous and non-invertible dissipative system.
- Crisis lifetime and measures of forbidden network and escaping hole show scaling behavior.
- The university of scaling exponents was discussed numerically.

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ABSTRACT

We investigate an integrate-and-fire model of an electronic relaxation oscillator, which can be described by the discontinuous and non-invertible composition of two mapping functions f_1 and f_2, with f_1 being dissipative. Depending on a control parameter d, f_2 can be conservative (for $d = d_c = 1$) or dissipative (for $d > d_c$). We find a kind of crisis, which is induced by the escape from a riddled-like attraction basin sea in the phase space. The averaged crisis transient lifetime ($\langle\tau\rangle$), the relative measure of the fat fractal forbidden network (η), and the measure of the escaping hole (Δ) show clear scaling behaviors: $\langle\tau\rangle \propto (d-d_c)^{-\gamma}$, $\eta \propto (d-d_c)^{\sigma}$, and $\Delta \propto (d-d_c)^{\alpha}$. Extending an argument by Jiang et al. (2004), we derive $\gamma = \sigma + \alpha$, which agrees well with numerical simulation data.

1. Introduction

In 1992–1994, scientists discovered that some nonlinear dynamical systems with a simple symmetry contain riddled basins [1,2]. These systems contain multiple attractors and the attraction basins of attractors are mixed such that for any initial condition approaching one attractor, there are initial conditions arbitrarily nearby that approach other attractors. Therefore, one cannot improve predictability by increasing the precision of the initial condition. In recent years, some new features have been revealed for such systems with riddled attraction basin [3–5]. In 2005, Lai et al. reported a new kind of mechanism, which can generate riddled basins in a non-symmetrical physical system [3]. The system is described by piecewise continuous and noninvertible links of two conservative mappings, f_1 and f_2. The phase space is divided into two

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distinct but complementary regions (R_1 and R_2), which are the domains of map functions f_1 and f_2, respectively. Furthermore, the map composed of f_1 and f_2 shows irreversibility, so that one phase point might have two preimages. Lai et al. discovered that the mechanism for generating the riddled basins is related to mixing different attraction basins on a fat-fractal set, which has been called the "forbidden region net". The forbidden region net contained attraction basins of all attractors in the system and the basin boundaries merely divided the fat-fractal sets on all scales [3].

The forbidden region may be expressed by $F_r = I \setminus [f_1(R_1) \bigcup f_2(R_2)]$, where I denotes the set of the whole phase space ($I = \{(x, y)|_{x\in[0,1],y\in[0,1]}\}$). The forward images of F_r seem also to be the forbidden region and should form a fat fractal network, however, some points there may have two backward images. One of them belongs to the forbidden region; the other is outside of it. Therefore the forbidden network should be the forward image set of F_r, excluding points which have two backward images. It can be expressed as $F_n = \bigcup_{j=1}^{\infty}[f^j(F_r)\setminus Q]$, where Q denotes the point sets which have two backward images. The boundaries of F_n have been found to be the forward images of the discontinuous borderline [3,6,7].

In 2006 Chao et al. [6] reported that a so-called "semi-dissipative" system, which is the concatenation of a conservative map f_1 and a dissipative map f_2, can also contain riddled basins induced by forbidden region network. Furthermore, the escaping from the riddled basins can produce a kind of crisis [6]. Compared with the findings in Ref. [3], the unique feature is that, inside the "riddled basin sea" in the phase space, very small classical dissipative attraction basins appeared in the vicinity of each periodic point. The dissipative basins served as the escape holes of the crisis.

In 2006, we proposed an integrate-and-fire model of an electronic relaxation oscillator [7,8], which can be described by the discontinuous and non-invertible composition of a dissipative mapping functions f_1 and a mapping function f_2 with a parameter d; when $d = d_c = 1, f_2$ is conservative; when $d > 1, f_2$ is dissipative. We found a kind of crisis, which is induced by an escape from a riddled-like attraction basin sea in the phase space. In this paper, we calculate the averaged crisis transient lifetime $\langle\tau\rangle$, the relative measure of fat fractal forbidden network η, and the measure of escaping hole Δ, finding that such quantities show clear scaling behaviors: $\langle\tau\rangle \propto (d - d_c)^{-\gamma}$, $\eta \propto (d - d_c)^{\sigma}$, and $\Delta \propto (d - d_c)^{\alpha}$. Extending an argument by Jiang, et al. [9], we derive $\gamma = \sigma + \alpha$, which agrees well with numerical simulation data.

2. The model system

The system discussed in Ref. [7] is an integrate-and-fire model of an electronic relaxation oscillator. Similar models are widely used in different scientific disciplines [10–13]. The maps describing the model is given as following [7]: When $x_n \in [0, x_0)$,

$$f_1 : \begin{cases} x_{n+1} = (1 + \frac{c}{b})x_n + y_{n+1} + \frac{a}{b}, \\ y_{n+1} = y_n + \frac{1}{b}\sin(2\pi x_n); \end{cases} \quad [mod.\ 1] \tag{1}$$

and when $x_n \in [x_0, 1]$,

$$f_2 : \begin{cases} x_{n+1} = (1 + C_1)x_n + y_{n+1} + \frac{a}{b} + C_2, \\ y_{n+1} = y_n + \frac{1}{b}\sin(2\pi x_n), \end{cases} \quad [mod.\ 1] \tag{2}$$

where

$$C_1 = \frac{c(1-d)}{b(1-x_0)} \text{ and } C_2 = \frac{c(d-x_0)}{b(1-x_0)}.$$

All the parameters, a, b, c, d, x_0 are constants determined by the circuit properties [7]. In the current study, we fix the parameters as $a = 2.0, b = 4.0, c = 0.3, x_0 = 0.1$, and consider d as a driving parameter.

It is well known that if the absolute value of the determinant of the Jacobian matrix of a map is one, the map is conservative; otherwise the map is dissipative [14]. Therefore submap f_1 is dissipative, and submap f_2 is conservative for $d = 1$ and dissipative for $d > 1$. The basic properties of the system undergo transitions as the parameter d passes the threshold, $d_c = 1$. When $d < d_c = 1.0$, the map function f_2 is also dissipative. Changing from $d = d_c = 1.0$ to $d < d_c = 1.0$ also corresponds to the transition from conservative map to dissipative map, which is not essentially different from the case when changing from $d = d_c$ to $d > d_c$ and is not discussed in this work.

3. The crisis

As shown in our previous work with the similar model [7], when $d = 1$, the phase plane is composed of three parts: (P_1) a stochastic web (the only chaotic orbit) formed by forward images of the borderline, $\{(x, y)|_{x=x_0}\}$ [3,6,7,15,16], (P_2) period-1 and period-6 elliptical islands, and (P_3) the fat fractal forbidden network (see the phase plane in Fig. 3 of Ref. [7]). Here the expression $\{(x, y)|_{x=x_0}\}$ represent the borderline of the domains of the two mapping function f_1 and f_2. The iterations starting from an initial value chosen in P_1 (P_2) are always moving on P_1 (P_2), without leaving; the iterations starting from an initial value chosen in P_3 will eventually enter P_1. When $d > 1$, P_1 suddenly changes to a transient web and P_2 becomes period-1 or period-6 point attractors (see Fig. 4 of Ref. [7] for the phase plane). The phase space occupied by P_1 and P_3 becomes