Physica A 500 (2018) 177–188

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Pricing and simulation for real estate index options: Radial basis point interpolation

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HIGHLIGHTS

- We model the real estate index options within the arbitrage-free framework.
- We consider the non-tradable and the mean-reverting characteristics of the index.
- We employ the radial basis point interpolation to price of real estate index options.
- Both European and American options contingent on real estate index can be priced.

ARTICLE INFO

Article history:
Received 26 June 2017
Received in revised form 20 December 2017

Keywords:
Real estate index options
Arbitrage-free approach
Option pricing
Radial basis point interpolation

ABSTRACT

This study employs the meshfree radial basis point interpolation (RBPI) for pricing real estate derivatives contingent on real estate index. This method combines radial and polynomial basis functions, which can guarantee the interpolation scheme with Kronecker property and effectively improve accuracy. An exponential change of variables, a mesh refinement algorithm and the Richardson extrapolation are employed in this study to implement the RBPI. Numerical results are presented to examine the computational efficiency and accuracy of our method.

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1. Introduction

The 2007 subprime mortgage crisis highlighted the importance of the real estate market to the global economy [1]. After this crisis, managing and hedging real estate market risks attracted significant attention. For example, Hinkelmann and Swidler [2] investigated whether the risk of residential real estate can be hedged with futures on other commodities and financial indices already traded in the market. The result indicated that house prices are difficult to be replicated with a portfolio of existing futures contracts. Therefore, it is necessary to establish real estate derivatives markets to eliminate the inefficiency of real estate markets and provide investors more financial instruments to hedge their real estate portfolios. In addition, real estate derivatives serve as gateways for investors to gain exposure to the price movements of real estate assets without buying or selling physical assets [3].

Real estate derivatives have been introduced in managing property price risk in the UK and USA markets. However, these financial instruments are still in a state of infancy [4]. In an early study, Shiller [5] proposed that two measurement problems may hinder the establishment and development of real estate derivative markets. The first problem is regard to the infrequency of underlying market price which may account for the slowness to develop derivatives markets. The second problem is that measurements are more on cash flows than on asset values or present values in many cases. Fabozzi et al. [6]

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https://doi.org/10.1016/j.physa.2018.02.135
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pointed out that the predictability of underlying asset returns makes it difficult to find counterparties in the real estate derivative markets, which will result in the illiquidity of real estate derivatives.

Several studies have focused on pricing real estate derivatives. Buttimer et al. [7] employed a bivariate binomial model to price the derivatives contingent on a real estate index and an interest rate. They found that their model can price a commercial real estate index total return swap leading to a positive but negligible swap spread price. Based on their model, Bjork and Clapham [8] proved that the theoretical price of the total return swap is equal to zero within an arbitrage-free framework. However, the aforementioned models ignored the fact that the real estate index is non-tradable. Taking the non-tradable characteristics of real estate index into consideration, Geltner and Fisher [9] employed an equilibrium model for the pricing of real estate forwards and total return swap contracts. Cao and Wei [10] proposed and implemented a different equilibrium valuation framework to analyze the valuation of housing index derivatives traded on the Chicago Mercantile Exchange (CME). Lizieri et al. [11] extended the model of Geltner and Fisher [9] by focusing on trading window effects resulting from market frictions in real estate. The results indicate that the observed spreads for total return swaps can be explained by the short-run momentum effects and imbalances of buyers and sellers. Van Bragt et al. [12] developed a risk-neutral valuation procedure for real estate derivatives by modeling the observed real estate index with an autoregressive model, and they derived closed-form pricing solutions for forwards, swaps, and European put and call options. Fabozzi et al. [1] proposed a pricing framework for real estate derivatives which can capture the econometric properties of real estate indices and complete the real estate market using real estate futures contracts. By assuming that the market price of risk is known, closed-form solutions can be obtained for main real estate derivatives, such as forwards, European options and total return swaps.

In option pricing literature, many numerical methods have been proposed for pricing options when closed-form solutions are difficult to obtain. The commonly used numerical approaches for pricing options are binomial or trinomial lattice methods [13–16], finite difference methods [17–19] and finite element or volume methods [20–22]. The common advantage of these numerical methods is that they can price derivatives under complex conditions, particularly when analytical solutions are difficult to obtain. In addition, numerous researchers have used meshfree methods to solve partial differential equations (PDEs). Meshfree methods do not require a pre-defined grid division manner, instead it only approximates finite points in the domain. Therefore, these methods are easier to implement with high accuracy than finite difference methods, finite element or volume methods [23]. In this study, we will employ a meshfree method, called the radial basis point interpolation (RBPI), which was originally proposed by Liu et al. [24] and extended by Rad et al. [25], to price real estate index options contingent on real estate index.

Point interpolation method (PIM) is one of the most common meshfree methods that recently attracted increasing attention. This method often employs two different kinds of interpolations, namely, polynomials basis point interpolation (PBPI) and RBPI [25]. PBPI is one of the earliest interpolation schemes with Kronecker property. However, the drawback of this method is that an inappropriate polynomial basis can lead to singularity [24]. Accordingly, RBPI can be used to overcome such drawback. For the RBPI method, several types of radial basis functions can be selected, such as, multiquadrics, inverse multiquadrics, Gaussians radial basis functions (RBFs), thin plate splines, and compactly supported radial basis functions (CSRBFs). It is necessary to choose appropriate shape parameters when implementing multiquadrics, inverse multiquadrics, and the Gaussians RBFs. If an inappropriate shape parameter is employed, then the results will be biased and the functions will no longer possess the Kronecker property [26–28]. The CSRBFs can reduce the original resultant full matrix to a sparse matrix. The operation of the banded matrix system reduces the ill-conditioning of the resultant coefficient matrix [29]. Therefore, this method is often used to solve large-scale problems. However, the thin plate splines do not involve any free shape parameter. Due to this advantage, Rad et al. [25] use the thin plate splines in their study.

The main purpose of this study is to extend the RBPI algorithm developed by Rad et al. [19] for pricing real estate index options contingent on real estate index. In this study, we consider the mean-reverting stochastic model for the real estate index in the study of Fabozzi et al. [1]. We obtain new PDEs of the real estate index options by using the arbitrage-free approach; RBPI is then employed to solve PDEs for European and American options.

This study contributes to the literature in the following aspects. First, we model real estate index options within the arbitrage-free framework and obtain new PDEs for real estate index options. Second, we extend the RBPI method to solve these PDEs. To the best of our knowledge, RBPI is a new method used in mathematical finance, which was extended by Rad et al. [25] to price options under the Black–Scholes model. Accordingly, this study extends this method to solve the model of Fabozzi et al. [1] for pricing real estate index options. Third, we extend the model of Fabozzi et al. [1] to price American options when closed-form solutions are difficult to obtain.

The rest of this study is organized as follows. Section 2 theoretically derives the equations for real estate index options. We introduce and extend the RBPI method to price real estate index options in Section 3. Section 4 presents the numerical analysis. Section 5 concludes this study.

2. Real estate index option model

2.1. Real estate index model

Real estate indices exhibit a positive autocorrelation in the short term and a negative autocorrelation in the long term [1]. Therefore, Fabozzi et al. [1] proposed a mean-reverting stochastic model to measure the real estate index movement.

$$dY_t = [\frac{d\psi_t}{dt} - \theta(Y_t - \psi_t)]dt + \sigma dW_t. \tag{1}$$

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