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Unidirectional Random Growth with Resetting

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Abstract

We review stochastic processes without detailed balance condition and derive their H-theorem. We obtain stationary distributions and investigate their stability in terms of generalized entropic distances beyond the Kullback-Leibler formula. A simple stochastic model with local growth rates and direct resetting to the ground state is investigated and applied to various networks, scientific citations and Facebook popularity, hadronic yields in high energy particle reactions, income and wealth distributions, biodiversity and settlement size distributions.

Keywords:
master equation, generalized entropic divergence, distributions in complex systems

1. Introduction

The challenge for physicists in taming complexity is to identify clear and simple models with possibly few ingredients and a great and rich reign of applicability. We have in mind achievements like the Ising model of magnetism [1], the Erdős–Rényi random graph model [2, 3], or the Landau Φ^4–theory for second order phase transitions [4]. These models have their beauty and usefulness not only in describing particular physical phenomena, but also in allowing for the gain of new fundamental insights. The Ising model led us to investigate critical behavior, the Erdős–Rényi graph to abundant research on path length and other optimization problems on random networks, and the Landau theory opened the door to study in a unified framework all types of phase transitions.

Master equations describing stochastic processes belong to a similar model class with a wide range of applicability to complex systems [5]. The question of stability of stationary solutions to such equations is recently connected to fundamental questions about the notion and correct mathematical treatment of entropy and entropic divergence. Most approaches using master equations contain equally growth and loss terms, and very often detailed balance condition is tacitly assumed. In several cases the stationary solution is presented, but the convergence rate to it is not elaborated.

In this paper we attempt an in-depth study of a particular class of stochastic processes: where the growth process dominates and only a very special transition to a ground state is allowed. This restriction immediately breaks the detailed balance condition. On the other hand such processes belong to the sample space reduction types in which a growing interest can be documented recently [6]. We give a derivation of a generalized H-theorem without relying on detailed balance for a subclass of master equations. The functional form of the dependence on the probability in describing its rate of change will be related to the construction of an information–theoretic entropic divergence formula seeked for in a trace form. Despite of its simplicity this approach offers a rich variety of complex behavior with corresponding probability distribution functions (PDF-s). In full agreement with the physicist's philosophy for handling complex systems the particular model we present here is based on only two dynamical ingredients: a growth rate and a reset rate.

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