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The behavioral implications of the bilateral gamma process

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A B S T R A C T

Bilateral gamma process is widely used in risk management and asset pricing. However the behavioral implications of this process remain unknown. This paper investigates this problem for the first time within the framework of Tauchen and Pitts (1983). With the assumption that there are two types of traders in the market, the optimistic and the pessimistic, we find the bilateral gamma process can be derived from Walrasian equilibrium. This finding establishes the microstructure foundations for the bilateral gamma process.

1. Introduction

The underlying process for the speculative asset price is of great importance in both asset pricing and risk management. The most celebrated processes include the random walk process of Bachelier [1] and the subordinate stochastic process of Clark [2]. In recent years more realistic stochastic processes have been developed by replacing the classical Brownian motion with Lévy processes, such as the generalized hyperbolic process of Barndorff-Nielsen [3], the variance-gamma process of Madan and Seneta [4], the CGMY process of Carr et al. [5]. A survey about Lévy processes used for applications to finance can be found in Schoutens [6].

The bilateral gamma (henceforth BG) process suggested by Kuchler and Tappe [7,8] also belongs to the family of Lévy processes. The BG process is defined as the difference of two independent gamma processes. Kuchler and Tappe [7,8] show that the BG process has very interesting properties: (1) it is self-decomposable, stable under convolution and have a simple cumulant generating function, (2) it is a finite-variation process making infinitely many jumps at each interval with positive length, and all the increments are bilateral gamma distributed. These properties make the BG process a potential candidate for the speculative asset price.

The BG process is now being widely used in both risk management and asset pricing. Kuchler and Tappe [7] find that the bilateral gamma density fits the empirical density of asset returns better than both the normal density and the variance-gamma density. Kuchler and Tappe [9] consider the option pricing problem with BG process in continuous time models. Bellini and Mercuri [10] extend the BG process to a conditional bilateral gamma process, in which the shape parameters of the bilateral gamma distribution have a Garch-like dynamics. Empirical studies performed on SPX options show that the conditional bilateral gamma process reports promising results compared with the models of Heston and Nandi [11], Christoffersen et al. [12] and with the dynamic variance gamma model of Mercuri and Bellini [13]. Kaishev [14] proposes a new Lévy process induced by Dirichlet splines which further generalizes the variance-gamma process and the BG process.

Despite its mathematical elegance and growing application, the behavioral implications of the BG process, to the best of our knowledge, remain unknown. This problem is important for at least two reasons: first, one of the main tasks in financial

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economics is trying to understand why and how asset price changes; second, unless the mechanism governing the BG process is fully understood, the process cannot be properly used in practice.

Intuitively, the process for speculative asset price is related to investors' behaviors. For example, if more traders become optimistic (pessimistic), the demand for the speculative asset will increase (decrease), which in turn will result in the rising (falling) of the asset price. With the assumption that there are two types of traders in the market, the optimistic and the pessimistic, we for the first time derive from Walrasian equilibrium that the speculative asset price can be summarized by a difference of two independent gamma processes, or the BG process.

This paper is organized as follows. Section 2 presents a short review on the BG process. Section 3 presents the equilibrium model from which the BG process is derived. Section 4 presents the conditional bilateral gamma process with some discussions. We conclude in Section 5.

2. A short review on bilateral gamma process

The BG process is defined as the difference of two independent gamma processes:

$$X = Y - Z \sim \Gamma(a, b, c, d) \tag{1}$$

$$Y \sim \Gamma(a, b) \tag{2}$$

$$Z \sim \Gamma(c, d), \tag{3}$$

where X is the BG distribution with $a, b, c, d > 0$, $\Gamma(\cdot)$ is the gamma distribution. The moment generating function of the BG distribution is given by

$$M_X(t) = \left(\frac{1}{1-bt}\right)^a \left(\frac{1}{1+dt}\right)^c, t \in \left(-\frac{1}{d}, \frac{1}{b}\right) \tag{4}$$

and its first four moments are given by

$$\begin{aligned}
\mu(X) &= ab - cd, \\
\sigma^2(X) &= ab^2 + cd^2, \\
\gamma(X) &= \frac{2(ab^3 - cd^3)}{\sqrt{(ab+cd)^3}}, \\
\kappa(X) &= 3 + \frac{6(ab^4 + cd^4)}{(ab+cd)^2},
\end{aligned}$$

where $\mu(\cdot)$, $\sigma^2(\cdot)$, $\gamma(\cdot)$ and $\kappa(\cdot)$ are respectively the mean, the variance, the skewness and the kurtosis. It is clear that the BG distribution is much more flexible for modeling the asset returns as it can easily reproduce the asymmetry and the high kurtosis which are well documented in returns. More properties about the BG process are available in Kuchler and Tappe, Kuchler and Tappe [7,8].

3. The model

Traders are heterogeneous [15–17], and can in general be classified as being either optimistic or pessimistic. Within the framework of Tauchen and Pitts [18] and with some modifications to their assumptions, we will present in the following section that the BG process can be derived from Walrasian equilibrium.

3.1. Equilibrium price

Suppose the market at time t consists of J_t active traders who take long and short positions in a speculative asset. The movement from the $t-1$st to the tth equilibrium is initiated by the arrival of new information to the market.

At the tth equilibrium the desired position of the jth trader is supposed to be determined by the following linear equation

$$Q_{t,j} = \alpha[P_{t,j} - P_t], (j = 1, 2, \ldots, J_t), \tag{5}$$

where $\alpha > 0$ is constant, $P_{t,j}$ is the jth trader's reservation price, and P_t is the current market price. Eq. (5) abstracts from the transaction costs and assumes that traders differ only in their reservation prices. A positive value for $Q_{t,j}$ represents a desired long position in the asset while a negative value represents a desired short position. These active traders have their reservation prices different from the current market quotation. The inter-trader difference in $P_{t,j}$ arises from different expectations about the future and from different needs to transfer risk through the market.

Equilibrium requires $\sum_{j=1}^{J_t} Q_{t,j} = 0$, which implies that the average of the reservation prices

$$P_t = \sum_{j=1}^{J_t} P_{t,j}/J_t \tag{6}$$

clears the market. Eq. (6) shows that the equilibrium price is determined by the average of the traders' reservation prices.