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Non-standard finite difference and Chebyshev collocation methods for solving fractional diffusion equation

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ABSTRACT

In this paper, a new numerical technique for solving the fractional order diffusion equation is introduced. This technique basically depends on the Non-Standard finite difference method (NSFD) and Chebyshev collocation method, where the fractional derivatives are described in terms of the Caputo sense. The Chebyshev collocation method with the (NSFD) method is used to convert the problem into a system of algebraic equations. These equations solved numerically using Newton's iteration method. The applicability, reliability, and efficiency of the presented technique are demonstrated through some given numerical examples.

1. Introduction

Fractional calculus has gained considerable popularity and importance due to its attractive applications as a new modeling tool in a variety of scientific and engineering fields, such as viscoelasticity [1], plasma turbulence models [2], fluid mechanics [3], electromagnetism [4], heat conduction [5], bioengineering [6] and optimal control [7]. In particular, modeling of anomalous diffusion in a specific type of porous medium is one of the most significant applications of fractional derivatives [8,9]. Moreover, there are other publications on this field dealing with various aspects in different ways and different numerical methods (see for instance, [10,11,8,12–14]).

The spectral methods have been intensively studied in the past decades Because of being extremely accurate [15]. The spectral methods lead to accurate approximate solutions because it employs linear combinations of orthogonal polynomials, as basis functions [15,16]. Mainly three types of spectral methods can be identified, tau [17], collocation [18–20], and Galerkin [21]. The spectral methods based on orthogonal systems, such as Chebyshev polynomials, Legendre polynomials, Bernstein polynomials and modified generalized Laguerre polynomials, are available for bounded and unbounded domains for the approximation of FDEs [22,21,7,10,23].

In 1989 the Non-Standard finite difference (NSFD) methods had they are born for solving the differential equations [24]. The basic rules and their application of these methods to some specific nonlinear equations appear in some papers [25,26]. In the last decades, applications of the NSFD for the discrete models have been constructed for a wide range of nonlinear dynamical systems such as the dynamics of HIV transmission [27], singular boundary value problems [28], modified linear heat/diffusion transport problems [29], and a generalized Nagumo reaction–diffusion model [30].

The fractional order diffusion equation considered one of the attractive concepts in the initial–boundary value problems in fractional. It has been used in modeling turbulent flow [2], viscoelastic materials [1], chaotic dynamics of classical

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conservative systems [5,9], applications in biology [6], in physics [8] and applications in other fields [15,4,13]. To obtain an analytical solution of this problems is extremely difficult thus many authors are seeking ways to numerically solve these problems. Therefore, there are appeared several numerical methods for solving the fractional diffusion equation such as in [23,31–33] the authors have used finite difference method, Also, Cui [34] is used compact finite difference scheme. On the other side, Sweilam and et al. [18,19] are used Chebyshev collocation method with finite difference method for solving the fractional diffusion equation. While Saadatmandi and Dehghan [17] are used operational matrix. Moreover, there are many methods for solving fractional order diffusion equation [35,31,32].

Our attention will be a focus on space fractional order diffusion equation of the form:

$$\frac{\partial u(x,t)}{\partial t} - r(x)\frac{\partial^{\alpha} u(x,t)}{\partial x^{\alpha}} = f(x,t),\ 0 < x < L,\ 0 < t \le T, \tag{1}$$

subject to the initial condition:

$$u(x,0) = u_0(x), \quad 0 < x < L, \tag{2}$$

and with the boundary conditions:

$$u(0,t) = q_0(t),\ u(L,t) = q_1(t), \quad 0 < t \le T \tag{3}$$

where the fractional derivative $\frac{\partial^{\alpha} u(x,t)}{\partial x^{\alpha}}$ in the Caputo sense, the parameter α refers to the fractional order of space derivative with $1 < \alpha \le 2$, $f(x,t)$ is the source term. The classical second-order diffusion equation is obtained from Eq. (1) at $\alpha = 2$:

$$\frac{\partial u(x,t)}{\partial t} - r(x)\frac{\partial^{2} u(x,t)}{\partial x^{2}} = f(x,t). \tag{4}$$

The main objective of the current work is to propose a new numerical technique for solving the space fractional order diffusion equation numerically. This technique is based on the Chebyshev collocation method in conjunction with the Non-Standard finite difference method. The approximate solution can be expressed as a series of shifted Chebyshev polynomials of the second kind in space with unknown coefficients in time. The (NSFD) is used to approximate the time term of the given equation. Using the introduced technique the diffusion equation is converted into a system of algebraic equations which can be solved numerically using Newton's iteration method.

The structure of this paper is arranged in the following way. We begin by recalling some facts from calculus that form the basis of our intended method with some properties of Chebyshev polynomials of the second kind. In Section 3, we introduce the fundamental theorem of the suggested method for solving the space fractional order diffusion equation in addition to the numerical scheme. Some numerical examples and Comparisons are presented in Section 4, to demonstrate the accuracy and efficiency of the suggested method. End the paper with conclusions Section.

2. Preliminaries and notations

Some necessary definitions and mathematical preliminaries of the fractional calculus theory that required for our subsequent development will be described in this section.

2.1. The Caputo fractional derivatives

Definition 1. The Caputo fractional derivative operator D^{α} of order α is defined as:

$$D^{\alpha} f(x) = \frac{1}{\Gamma(m-\alpha)}\int_0^x \frac{f^{(m)}(t)}{(x-t)^{\alpha-m+1}}dt, \quad \alpha > 0, \tag{5}$$

where $m-1 < \alpha \le m,\ m \in \mathbb{N},\ \Gamma(\cdot)$ is the Gamma function & $x > 0$.

The Caputo fractional order derivative operator have property of linearity as the integer order operator of differentiation:

$$D^{\alpha}(\lambda f(x) + \mu g(x)) = \lambda D^{\alpha} f(x) + \mu D^{\alpha} g(x), \tag{6}$$

where λ and μ are constants.

For the Caputo fractional order derivative operator, we can obtain the result:

$$D^{\alpha} K = 0, \quad K \text{ is a constant}, \tag{7}$$

$$D^{\alpha} x^n = \begin{cases} 0 & \text{if} \quad n \in \{0,1,2,\ldots,\lceil\alpha\rceil - 1\}, \\ \dfrac{\Gamma(n+1)}{\Gamma(n+1-\alpha)} x^{(n-\alpha)} & \text{if} \quad n \in \mathbb{N} \wedge n \ge \lceil\alpha\rceil \\ & \text{or} \quad n \notin \mathbb{N} \wedge n > \lceil\alpha\rceil - 1, \end{cases} \tag{8}$$

where the smallest integer greater than or equal to α is denoted by the function $\lceil\alpha\rceil$. Also, the Caputo differential operator is the same as a usual integer order differential operator in case of $\alpha \in \mathbb{N}$. For more details on fractional derivatives definitions and its properties see [13].