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Quantifying phase synchronization using instances of Hilbert phase slips

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HIGHLIGHTS

- Proposes a novel approach to quantify phase synchrony between two signals.
- The approach requires not all non-invertible transformations in the original method.
- The approach performs equally well as the original approach.

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ABSTRACT

We propose to quantify phase synchronization between two signals, $x(t)$ and $y(t)$, by calculating variance in the Hilbert phase of $y(t)$ at instances of phase slips exhibited by $x(t)$. The proposed approach is tested on numerically simulated coupled chaotic Roessler systems and second order autoregressive processes. Furthermore we compare the performance of the proposed and original approaches using uterine electromyogram signals and show that both approaches yield consistent results A standard phase synchronization approach, which involves unwrapping the Hilbert phases ($\phi_1(t)$ and $\phi_2(t)$) of the two signals and analyzing the variance in the $|n \cdot \phi_1(t) - m \cdot \phi_2(t)|$, mod 2π, (n and m are integers), was used for comparison. The synchronization indexes obtained from the proposed approach and the standard approach agree reasonably well in all of the systems studied in this work. Our results indicate that the proposed approach, unlike the traditional approach, does not require the non-invertible transformations — unwrapping of the phases and calculation of mod 2π and it can be used to reliably to quantify phase synchrony between two signals.

1. Introduction

Phase synchronization is a technique that has been used widely to quantify synchrony between two signals [1–4]. The mathematical principle behind phase synchronization analysis is that two signals in synchrony will maintain a constant phase difference. Thus, a straightforward approach to quantify phase synchronization is to calculate instantaneous phases of the two signals and assess the constancy of the phase difference between the two signals. An instantaneous phase of a signal can be calculated using either a wavelet transform [5] or a Hilbert transform [2]; however, the latter is more commonly used in phase synchronization analysis. The Hilbert phase of a signal exhibits slips (transitions from $-\pi$ to π or vice versa) when the signal completes a cycle [6]. Prior to the calculation of the phase difference, a transformation – namely, a phase unwrapping – is applied to the phases and this transformation adds a value of 2π when the phase exhibits

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a slip. The unwrapped phase forms a straight line with slope $\omega \cdot t$, with ω being the frequency of the signal and t being the time. The phase difference between the two signals is calculated as the digital subtraction of an unwrapped phase of one signal from the other. The variance of the phase difference is often assessed using the Shannon entropy [2,7] or the first mode of the Fourier transform [1,8]. In the case of weak synchrony, the phase difference might exhibit a trend and this trend would impede the quantification of the phase synchronization. To address this limitation, one can calculate cyclic relative phases using a modulo function. Alternatively, a recently proposed gradient approach can also be used to quantify weak synchrony between two signals [9]. Thus, quantification of phase synchronization involves applying Hilbert transform, phase unwrapping, and calculating the modulo function of the phase differences. In this work, we attempt to minimize the use of the number of transformations involved in the phase synchronization quantification without compromising the reliability. In this letter, we propose utilizing instances at which one of the signals exhibited slips and assessing the variance of the Hilbert phases of the second signal at these instances. We apply the proposed approach to numerically simulated data using two coupled chaotic Roessler systems and a second order autoregressive process; Finally, we demonstrate the application of the proposed and original approaches in quantifying synchronization in uterine contraction signals measured using transabdominal electromyogram from pregnant women and show that both approaches yield comparable results. In all analysis, the performance of proposed approach was compared with that of the original approach.

2. Methods

Let $x(n)$ and $y(n)$ be two signals measured as a continuous function of time with n being the sample number. The Hilbert transform $h_x(t)$ of $x(t)$ is calculated as follows:$h_x(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$, with P being Cauchy's Principal value, which ensures that nodal points are avoided within the interval of the integral and $t = n \cdot s$, where s is the sample rate. The signal and its Hilbert transform can be represented as a complex analytic form as follows: $a_x(n) = x(n) + i \cdot h_x(n)$, with $i = \sqrt{-1}$. All analyses were carried out in MATLAB (Mathworks Inc, NA, MA). The 'Hilbert' function in MATLAB was used to calculate the Hilbert transform of the signals, which directly calculated the complex analytic function. The instantaneous phase $\phi_x(n)$ is then calculated using the arctangent transform of the complex analytic function. In a similar way, we calculated the instantaneous phase $\phi_y(n)$ of the signal $y(n)$.

2.1. Original approach

The phases are unwrapped using the 'unwrap' function in MATLAB, which eliminates the jumps in the phases. Let us denote the unwrapped phases as $\phi'(n)$. The relative phase difference $\Delta\phi(n)$ is calculated as $\Delta\phi(n) = \lfloor m \cdot \phi'_x(n) - n \cdot \phi'_y(n) \rfloor$, where m and n are integers. To calculate the phase synchronization index, the relative phase difference is converted into relative cyclic phase differences as $\Delta\varphi(n) = \Delta\phi(n) \bmod 2\pi$, where mod$(\cdot)$ is a modulo function. To this end, the phase synchronization index ρ is defined using the first mode of the Fourier transform as $\rho = \sqrt{\langle \cos(\Delta\varphi(n)) \rangle^2 + \langle \sin(\Delta\varphi(n)) \rangle^2}$, with $\langle \cdot \rangle$ being the arithmetic average taken over all samples. The phase synchronization index takes on a value of one in cases of perfect synchrony, and a value of zero in cases of asynchrony. The above analysis is carried out for different values of n and m. The two signals $x(n)$ and $y(n)$ are said to be $n : m$ synchronized if m and n maximized ρ.

2.2. Proposed approach

2.2.1. Procedure

We identify the instances of phase slips in $\phi_x(n)$, as discussed in our previous study [6], and denote those instances as ω_i. Using ϕ_y at these locations, we calculate the phase synchronization index as follows: $\gamma = \sqrt{\left\langle \cos\left(\phi_y(k)\right) \right\rangle^2 + \left\langle \sin\left(\phi_y(k)\right) \right\rangle^2}$, $\forall k \in \omega$. Let the number of phase slips in $x(n)$ and $y(n)$ be n_x and n_y. Calculate the two ratios $R_1 = nint\left\{n_x/n_y\right\}$ and $R_2 = nint\left\{n_y/n_x\right\}$, where *nint* denotes the nearest integer, which is accomplished using the "round" function in MATLAB. To detect the $n : m$ coupling we consider the following three scenarios:

Case 1 $R_1 = R_2 = 1$, it implies a $1 : 1$ synchrony between the two signals, and for this scenario we will follow the above procedure.

Case 2 $R_1 > 1$, implies that $x(t)$ oscillates R_1 times faster than $y(t)$. To match the frequency of $y(t)$ and to quantify the synchrony correctly, we downsample ω_i by a factor of R_1 and use the above procedure. In this case, it implies a $1 : R_1$ synchrony between the two signals.

Case 3 $R_2 > 1$, implies that $y(t)$ oscillates R_2 times faster than $x(t)$. To match the frequency of $x(t)$ and to quantify synchrony correctly, use $\phi_y(t)$ only at instances of R_1. In this case, it implies an $R_2 : 1$ synchrony between the two signals.

2.3. Assessing statistical significance of the phase synchronization index

We have assessed the statistical significance of the phase synchronization index obtained using original and modified approaches using a time-shifted analysis approach. In this approach, we held the first signal constant and generated 100 different time-shifted versions of the second signal. The first realization was obtained by shifting 1-s of the second signal. For the subsequent realizations, the shift period (samples) was obtained as follows: $\tau = \left[\frac{N-sf}{99}\right]$, where *sf* is the sampling