Physica A 499 (2018) 362–375

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Kinetic models for goods exchange in a multi-agent market

Carlo Brugna, Giuseppe Toscani *

Department of Mathematics, University of Pavia, via Ferrata 1, 27100 Pavia, Italy

HIGHLIGHTS

- Price formation of a good in presence of two classes of agents: dealers and speculators.
- Both classes adopt the same strategy, but speculators play on quantities of goods to be exchanged.
- Construction of kinetic models for the price evolution along classical micro economic rules.
- Numerical experiments validate the strategy of speculators.

ARTICLE INFO

Article history:
Received 5 October 2017
Received in revised form 7 December 2017
Available online 16 February 2018

Keywords:
Wealth and income distributions
Kinetic models
Boltzmann equation

ABSTRACT

In this paper we introduce a system of kinetic equations describing an exchange market consisting of two populations of agents (dealers and speculators) expressing the same preferences for two goods, but applying different strategies in their exchanges. Similarly to the model proposed in Toscani et al. (2013), we describe the trading of the goods by means of some fundamental rules in price theory, in particular by using Cobb–Douglas utility functions for the exchange. The strategy of the speculators is to recover maximal utility from the trade by suitably acting on the percentage of goods which are exchanged. This microscopic description leads to a system of linear Boltzmann-type equations for the probability distributions of the goods on the two populations, in which the post-interaction variables depend from the pre-interaction ones in terms of the mean quantities of the goods present in the market. In this case, it is shown analytically that the strategy of the speculators can drive the price of the two goods towards a zone in which there is a branded utility for their group. Also, according to Toscani et al. (2013), the general system of nonlinear kinetic equations of Boltzmann type for the probability distributions of the goods on the two populations is described in details. Numerical experiments then show how the policy of speculators can modify the final price of goods in this nonlinear setting.

1. Introduction

In recent years, there has been an increasing interest in developing kinetic models able to describe price formation in a multi-agent society, by resorting to methods typical of statistical mechanics [1,2]. In [3] Cordier, Pareschi and Piatecki introduce a kinetic description of the behavior of a simple financial market consisting of a population of agents where each agent can choose to invest between a stock and a bond. In this case, the variation of density is derived starting from the microscopic model for price formation introduced in [4,5], usually known as Levy–Levy–Solomon model. The kinetic model proposed in [3] attempts to join to simple financial rules a kinetic equation of Boltzmann type, able to describe a complex behavior that could then mimic the market and explain the price formation mechanism.

* Corresponding author.
E-mail addresses: carlo.brugna@gmail.com (C. Brugna), giuseppe.toscani@unipv.it (G. Toscani).

https://doi.org/10.1016/j.physa.2018.02.070
0378-4371/

A further example of coupling wealth with behavioral aspects has been proposed in [6]. This research studies a relatively simple kinetic model for a financial market characterized by a single stock or good and an interplay between two different trader populations, chartists and fundamentalists, which determine the price dynamics of the stock. The model has been inspired by the microscopic Lux–Marchesi model [7,8]. The financial rules depend on the opinion of traders through a kinetic model of opinion formation recently introduced in [9]. A related model has been developed in [10], by allowing the opinion variable, which is mainly responsible of the trading, to be strictly connected to price acceleration.

Also, the importance of the personal knowledge of agents has been recently investigated in [11], in order to outline how wealth inequality could depend on knowledge distribution in a population.

In a recent paper, driven by the assumption that people trade to improve its utility, we coupled the methods of statistical mechanics and kinetic theory with some principle of price theory in microeconomics [12]. This research followed other attempts to investigate price formation on a population of agents by means of Cobb–Douglas utility functions, first considered in [12]. In [13] it was shown that a Cobb–Douglas utility minimization, subject to money and commodity constraints, leads to the microscopic kinetic exchange considered in [14,15], namely to the binary kinetic exchange with uniform saving propensity of agents. An exhaustive discussion on the relationships between utility functions and related kinetic models can be found in Chapter 6 of the recent book [16]. In [12], we studied kinetic models in which the binary interactions among agents follow the rule furnished by the Edgeworth box [17], which is frequently used in general equilibrium theory.

Edgeworth box can fruitfully be applied in presence of an agent-based system in which agents possess a finite number of goods of $n \geq 2$ different types. Inspired by this mechanism of increasing utility and competitive equilibrium, in [12] was introduced and studied a kinetic equation of Boltzmann type for the evolution of the distribution density of the quantities of two goods in a system of agents. The exchange rule based on the Edgeworth box idea leads to a highly nonlinear binary interaction, which is difficult to handle, if not numerically.

For this reason, in [12] was considered a suitable linear Boltzmann equation, obtained by allowing the agent to interact (according to Edgeworth box), simultaneously with a sufficiently high number of agents in the market. For this model, it was shown that this linear equation has a unique solution, and the steady states are concentrated along a well-defined line (the price line).

Motivated by the interesting outcomes of the kinetic model based on binary trades driven by the Edgeworth box exchange, and taking into account the intrinsic interest of studying different types of populations which behave differently with the aim of getting maximum utility [6–8,11], in what follows we will introduce and discuss a kinetic description of a multi-agent system composed by two populations which interact according to the principle to get maximum utility, but allowing one of the two populations to exchange goods, by using only a part of them in the cross exchange, with the aim of getting a better profit from this strategy. In analogy with Lux–Marchesi description [7,8] we will define this population as the population of *speculators*, by leaving the name of *dealers* to the other one.

Applying simple principles of micro-economy [18], we first derive in Section 3 a linear system of kinetic equations of Boltzmann type, which describes the evolution of the quantities of goods in the two populations. It is shown that the evolution in time of the mean price obeys a nonlinear law, which in some cases can be explicitly given, to show that the speculators can effectively obtain a net wealth gain from their strategy. Then, in Section 4 we will introduce a system of nonlinear kinetic equations, similar to the one considered in [12], which is able do describe the action of a group of speculators in a market of dealers. Numerical experiments, performed in Section 5, enlighten the possible outcomes of the various strategies.

2. The basic model

As discussed in the introduction, most of the existing kinetic models for wealth distribution are based on rigid assumptions which, if on one hand can be shared, from the other hand are not deeply related to economic principles, like price theory [19]. The aim of this Section is to introduce a framework for trades, which is derived directly from the basic principles of economy [18] (cf. also [4]).

Individuals exchange goods. The benefits they receive depend on how much they exchange and on what terms. Price theory tries to give an answer to this fundamental question.

For simplicity, let us start by considering a market with a number N of agents which possess goods of two different types, we denote by X and Y. At the starting time, agents (indexed by k) possess certain amounts $x_k = x_k(0)$ of good X and $y_k = y_k(0)$ of good Y. While it is clear that x_k and y_k belong to $\mathbb{N}_+$, to avoid inessential difficulties, and without loss of generality, we will always consider these numbers as positive real numbers. The total number of each good in the disposal of agents is given by

$$\sum_{k=1}^{N} x_k = M_x, \quad \sum_{k=1}^{N} y_k = M_y. \tag{2.1}$$

We assume moreover that the market is closed, so that the total quantity of goods to be exchanged remains fixed in time. At fixed intervals of time of length Δt, agents exchange parts of their goods following a certain strategy. In view of these exchanges, agents hold at times t amounts of good X and Y, respectively denoted by $x_k(t)$ and $y_k(t)$. By virtue of (2.1), for