Physica A 501 (2018) 238–247

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Cluster synchronization transmission of different external signals in discrete uncertain network

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HIGHLIGHTS

- We research cluster synchronization transmission of different external signals in discrete uncertain network.
- The network controller and the identification law of uncertain adjustment parameter are designed.
- The network nodes in each cluster and the transmitted external signal can be different.
- The clustering topologies, the cluster number and the node number in each cluster can be freely chosen.

ARTICLE INFO

Article history:
Received 15 September 2017
Received in revised form 19 November 2017
Available online 15 March 2018

Keywords:
Cluster synchronization
Discrete uncertain network
Parameter identification
Lyapunov theorem

ABSTRACT

We research cluster synchronization transmissions of different external signals in discrete uncertain network. Based on the Lyapunov theorem, the network controller and the identification law of uncertain adjustment parameter are designed, and they are efficiently used to achieve the cluster synchronization and the identification of uncertain adjustment parameter. In our technical scheme, the network nodes in each cluster and the transmitted external signal can be different, and they allow the presence of uncertain parameters in the network. Especially, we are free to choose the clustering topologies, the cluster number and the node number in each cluster.

1. Introduction

In the network dynamic field, synchronization behavior is one of the main research directions. Due to its important potential applications in real-world, synchronization in complex networks has attracted great attention [1–6]. Pecora and Carroll have done pioneering work in the field of network synchronization, and they used the master stability function criterion to complete the synchronization of the regular network [7]. After that, the establishments of the small world [8] and scale-free network [9] make it possible to study the synchronization of irregular networks [10–14]. In recent years, a large number of literatures reported the research results of network synchronization. As a result, different types of network synchronization have been put forward, including complete synchronization [15–17], phase synchronization [18,19], projective synchronization [20,21] and cluster synchronization [22,23], etc.

Among various synchronization types, the cluster synchronization of the network has gradually aroused people's wide attention. The cluster synchronization of the network means that the dynamical nodes within each cluster synchronize with each other, but without synchronization appearing between any two nodes from different clusters. It is obvious that the clustering synchronization of the network is of great practical value, which is worthy of further research. For this, Tang

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https://doi.org/10.1016/j.physa.2018.02.156
0378-4371/

et al. investigated cluster synchronization for dynamical networks consisting of delayed nonlinear coupling Lur'e systems with identical and distinct communities [24]. Rakkiyappan and Sakthivel researched the cluster synchronization problem for T–S fuzzy complex networks with probabilistic time-varying delays and proposed the pinning control strategy [25]. Sorrentino et al. described a method to find and analyze all of the possible cluster synchronization patterns in a Laplacian-coupled network by applying method of computational group theory on dynamically equivalent networks. And they presented a general technique to evaluate the stability of each dynamically valid cluster synchronization pattern [26]. Vahedi and Noorani investigated the cluster modified projective synchronization between two topologically distinct community networks [27]. Jalan et al. researched the evolutions of coupled chaotic dynamics of networks and investigated the role of degree–degree correlation in the networks' cluster synchronizability. They found that the increase of the disassortativity can lead to an increase or a decrease in the cluster synchronizability depending on the degree distribution and average connectivity of the network [28]. Lü et al. proposed a new technique for the cluster synchronization between uncertain networks with different dynamics. Based on the Lyapunov theorem and Lipschitz condition, the network controllers and the identification laws of uncertain parameters are designed, and they are efficiently used to achieve the cluster synchronization and the identification of uncertain parameters [29].

Up to now, the research on cluster synchronization of the network is focused mainly on the situation that the network nodes in each cluster and the transmitted external signal are the same. But in practical applications, there are many phenomena that the network nodes in each cluster and the transmitted external signal are different. At present, there are few reports about this kind of research. In this work, we research cluster synchronization transmissions of different external signals in discrete uncertain network. Based on the Lyapunov theorem, the network controller and the identification law of uncertain adjustment parameter are designed and they are efficiently used to achieve the cluster synchronization and the identification of uncertain adjustment parameter. In our technical scheme, the network nodes in each cluster and the transmitted external signal can be different and they allow the presence of uncertain parameter in the network. Especially, we are free to choose the clustering topologies, the cluster number and the node number in each cluster.

The rest of the work is organized as follows. In Section 2, we first present the mathematical model and some preliminaries. In Section 3, main results, including designs of network controller and identification law of uncertain adjustment parameter, are presented. In Section 4, numerical simulation is given to verify the theoretical results. Finally, we make the conclusion in Section 5.

2. Problem description

Considering an arbitrary discrete system with spatiotemporal behavior

$$x(m,n+1)=F(x(m,n),\theta) \tag{1}$$

where m and n denote discrete space and time, respectively. $x(m,n)\in R^s$ is state variable of system and $F: R^s\rightarrow R^s$. θ is the parameter.

N-discrete Eqs. (1) are selected as the nodes to construct a complex network and it is divided into p clusters. Then, the state equation of the network node can be expressed by

$$x_i(m,n+1)=F(x_i(m,n),\theta_i)+\xi_i\sum_{j=1}^{N}c_{ij}x_j(m,n)+u_i(m,n)\quad i\in\Omega \tag{2}$$

here there are not any limitations for the division of the clusters, the number of nodes in each cluster and the connection between nodes. If k_l represents the number of nodes in the lth cluster, $\Omega=\{k_1+k_2+\cdots+k_{l-1}+1,\ldots,k_1+k_2+\cdots+k_{l-1}+k_l\}$ refers to the index set of all the nodes in the lth cluster, where $l=1,2,\ldots,p$ and $k_1+k_2+\cdots+k_p=N$. ξ_i is the coupling strength between the network nodes and c_{ij} is the coupling matrix element that represents the network topology structure. If the connection between node i and node j exists, $c_{ij}\neq 0$; otherwise, $c_{ij}=0$. $u_i(m,n)$ is the network controller.

We assume that the dynamical equation of the transmitted external signals is

$$x_l(m,n+1)=G_l(x_l(m,n),\omega_l)\quad (l=1,2,\ldots,p) \tag{3}$$

where ω_l is the parameter.

For $i\in\Omega$, the synchronization error is defined as

$$e_i(m,n)=x_i(m,n)-x_l(m,n) \tag{4}$$

and then the time derivative of error can be obtained by

$$\begin{aligned} e_i(m,n+1) &= x_i(m,n+1)-x_l(m,n+1)\\ &= F(x_i(m,n),\theta_i)-G_l(x_l(m,n),\omega_l)+\xi_i\sum_{j=1}^{N}c_{ij}x_j(m,n)+u_i(m,n). \end{aligned} \tag{5}$$

Definition 1. Cluster synchronization transmission of different external signals in discrete uncertain network is realized if $\lim_{n\to\infty}|x_i(m,n)-x_l(m,n)|=0$.