Physica A 499 (2018) 142–147

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Dynamic fractals in spatial evolutionary games

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HIGHLIGHTS

- The steady state of a spatial game features a series of dynamic regimes.
- Transitions between regimes do not fit into the standard paradigm of phase transitions.
- Interfaces are random fractals which are space filling in the thermodynamic limit.

ARTICLE INFO

Article history:
Received 29 November 2017
Received in revised form 24 January 2018
Available online 10 February 2018

Keywords:
Cluster interface
Spatial games
Dynamic fractals
Prisoner's Dilemma

ABSTRACT

We investigate critical properties of a spatial evolutionary game based on the Prisoner's Dilemma. Simulations demonstrate a jump in the component densities accompanied by drastic changes in average sizes of the component clusters. We argue that the cluster boundary is a random fractal. Our simulations are consistent with the fractal dimension of the boundary being equal to 2, and the cluster boundaries are hence asymptotically space filling as the system size increases.

1. Introduction

The surge of interest in game theory can be traced to the seminal works of John Nash in the middle of the 20th century. The main subject of classical game theory is finding the optimal strategy in games between two or more individuals (players or agents), where each individual has several possible behaviors. A repeated game is a situation where the same agents play the game with the same rules multiple times. Rational behavior of an agent then evolves with time based on the memory of past encounters. An agent's strategy thus evolves, the so-called evolution of cooperation [1,2].

A prototypical model in game theory is the so-called Prisoner's Dilemma, played by two agents in discrete time steps. In each round of the game, each agent uses one of two possible strategies, *cooperate* $\mathcal{C}$ or *defect* $\mathcal{D}$, and receives a payoff that depends on the strategies of the agent and its opponent [3].

Evolutionary game theory (see, e.g., [4–7] and the references therein) investigates the behavior of large populations, where a macroscopic number of agents use a finite number of strategies. While classical game theory deals with individual agents, evolutionary game theory focuses on the winning strategies themselves rather than individuals. Spatial evolutionary games are played with agents arranged in some spatial structures and interacting with other agents in their immediate neighborhoods. Various geometries have been explored, including regular grids [8,9], random graphs and small world networks [10], and evolving random graphs [11,12].

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https://doi.org/10.1016/j.physa.2018.02.007
0378-4371/

The spatial arrangement of agents yields emergent geometric structures—groups of agents who synchronize their behaviors with their neighbors and compete with other groups. The temporal evolution of these geometric structures can be highly nontrivial.

In this paper, we study a simple version of an evolutionary game based on the Prisoner's Dilemma [8,9]. The game is deterministic, and the time evolution is governed by a single parameter, the payoff ratio. Although the local rules are apparently simple, the steady state of the game features a series of very different dynamic regimes separated by sharp transitions. We characterize the geometric properties of the emergent structures across transitions.

We obtained several results that might be surprising for the statistical physics community. We found that transitions between steady states of the structures do not fit into a traditional classification of critical phenomena. The transitions are sharp but are not similar to first-order thermodynamic transitions [13]. Clusters of agents with similar strategies do percolate from boundary to boundary of the finite systems investigated. Cluster mass is finite at both sides of the transition, and it is similar to the first order phase transition. It is known that cluster interface scales as the cluster surface in these case, and it is not fractal. In our case cluster boundaries look irregular and demonstrate fractal behavior, which is similar to the percolation clusters in thermodynamic equilibrium [14,15]. We measure dimension of the cluster interface and find that it equals 2 asymptotically with the lattice size.

2. Rules of the game

Following Refs. [8,9], we define the game rules as follows: L^2 agents are arranged on an $L \times L$ rectangular grid in two dimensions. The game is globally synchronous and is played in discrete time steps. At each time step, an agent interacts with its eight neighbors (the chess king's moves) and itself.[1] The total score of an agent in a round is the sum of the payoffs of all nine games played in the current round. When all pairwise games are played and all payoffs are known, agents change their strategies for the next round. Various adaptation behaviors are possible. We use the simplest case of maximally opportunistic agents with a short memory: at each time step, an agent adopts the strategy with the maximum payoff among itself and its opponents in the preceding round. For two agents, this strategy is trivial. It becomes more interesting when the maximally opportunistic adaptation is used in a spatial evolutionary context.

The payoff an agent receives in an elementary game depends on the strategies of the agent and its opponent. We use the following payoff structure [8,9]: (i) If both agents defect, they receive nothing. (ii) If both agents cooperate, each of them receives a payoff of S, which we set to $S = 1$ without loss of generality. (iii) In the interaction of $\mathcal{C}$ and $\mathcal{D}$, the defector receives a payoff $T > S$ and the cooperator receives zero. The payoff structure hence depends on only one parameter, the payoff ratio $b = T/S$.

The spatial game can obviously be described as a cellular automaton with a particular set of transition rules, but the description in terms of cellular automata turns out to be very complex: the state of an agent in the next round depends on the payoffs of its neighbors, which in turn depend on their neighbors. Because 25 agents are relevant, the transition table size is 2^{25}, in contrast to the transition matrix for Conway's Game of Life, which has $2^9 = 512$ rules.

3. Results and discussion

3.1. Quantitative analysis

The game is deterministic, and the full time evolution is completely defined by the initial conditions (the spatial distribution of strategies $\mathcal{C}$ and $\mathcal{D}$ at $t = 0$) and the value of the payoff parameter b. The discrete structure of the payoffs leads to a series of very different dynamic regimes separated by sharp transitions at special values of b. Moreover, for fixed initial conditions, the dynamics is exactly identical for all values of b between these transition points. Statistical fluctuations enter via our use of random, unstructured initial conditions: physical observables are calculated as averages over both time evolution in the steady state (which is deterministic given initial conditions) and the ensemble average over a set of realizations of initial conditions (where the steady states are equivalent in the statistical sense).

It is instructive to consider the time evolution of small local objects, i.e., clusters of one strategy embedded into a sea of the other strategy. For $b < 1$, defectors always lose. For $b > 3$, cooperators unconditionally win. For $1 < b < 9/5$, a zoo of various small objects (gliders, rotators, etc.) are possible. Small clusters of $\mathcal{D}$ remain small and large clusters of $\mathcal{D}$ shrink. Conversely, for $b > 9/5$, a 2×2 or larger cluster of $\mathcal{D}$ grows. The situation is reversed for defectors: a 2×2 cluster of $\mathcal{C}$ grows for $b < 2$, while a large cluster of $\mathcal{C}$ shrinks for $b > 2$.

Therefore, $9/5 < b < 2$ is the fierce competition regime where clusters of $\mathcal{C}$ can grow in regions of $\mathcal{D}$ and vice versa. Starting from a single defector in a center of a large game field, the steady state is a dynamic fractal with a well-defined average density of $\mathcal{D}$, the "evolutionary kaleidoscope" in Ref. [9].

[1] The presence of self-interaction can be motivated by considering an agent to represent a group of individuals. Self-interaction then encapsulates some internal dynamics of this group. The qualitative features of the model are independent of the inclusion of self-interaction.