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A key heterogeneous structure of fractal networks based on inverse renormalization scheme

Yanan Bai [a], Ning Huang [a,b,*], Lina Sun [a]

[a] School of Reliability and Systems Engineering, Beihang University, Beijing, 100191, PR China
[b] Key Laboratory of Science & Technology on Reliability & Environmental Engineering, Beihang University, Beijing, 100191, PR China

HIGHLIGHTS

- The primitive structure is a key heterogeneous structure of fractal networks.
- A degree variance index is proposed to measure the dispersion of nodes degree.
- The efficiency of generated networks is positively correlated with degree variance.

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ABSTRACT

Self-similarity property of complex networks was found by the application of renormalization group theory. Based on this theory, network topologies can be classified into universality classes in the space of configurations. In return, through inverse renormalization scheme, a given primitive structure can grow into a pure fractal network, then adding different types of shortcuts, it exhibits different characteristics of complex networks. However, the effect of primitive structure on networks structural property has received less attention. In this paper, we introduce a degree variance index to measure the dispersion of nodes degree in the primitive structure, and investigate the effect of the primitive structure on network structural property quantified by network efficiency. Numerical simulations and theoretical analysis show a primitive structure is a key heterogeneous structure of generated networks based on inverse renormalization scheme, whether or not adding shortcuts, and the network efficiency is positively correlated with degree variance of the primitive structure.

1. Introduction

A network model consisting of nodes and edges is the foundation of study for complex systems. Many network models were proposed to generate networks with one or more statistical and topological characteristics, and then to analyze network behaviors [1–8]. Even the networks have same statistical characteristics, they still exhibit various structural property. Based on the discovery of statistical and topological properties of complex networks, many stochastic network models [9–12] were proposed to generate networks with small world effect [9] or scale-free feature [10]. To better understand and investigate analytically the topological property of complex networks, Barabasi et al. [13] proposed a deterministic network model, and then deterministic network models developed into fractal growth models [14–16] (pseudo-fractal networks [17–20]).

* Correspondence to: School of Reliability and Systems Engineering, Beihang University, No.37 Xueyuan Road, Haidian District, Beijing, 100191, PR China.
E-mail address: hn@buaa.edu.cn (N. Huang).

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Since the finding of self-similarity property [21] in complex networks, many researches focus on fractal networks, such as fractal property [22–26], fractal network model [27–31], multi-fractality [32–34], and so on. For fractal networks, Kim et al. [29] showed that they comprise a skeleton and shortcuts beyond the network model consisting of nodes and edges, and the skeleton exhibits fractal scaling similar to that of the network. As the scaffold for fractality of networks, the skeleton can be generated by the branching process [29] starting from a root node or the inverse renormalization scheme [35] starting from a primitive structure. According to Ref. [35], based on the renormalization group technique that coarse-grains the network into boxes containing nodes within a given lateral size, we can find the distribution of shortcuts overlaying a skeleton. When the exponent of the shortcuts $\alpha > 2d_B$, the network structure exhibits the pure fractal property. On the other hand, when $\alpha \leq 2d_B$, the network topology behaves as a small world at large sizes of box, and the network displays the small-world property at all sizes of box for small values of α. In return, a primitive structure can grow into a fractal network by inverse renormalization scheme [27], and then, the network evolves into a small-world network through adding corresponding type of shortcuts [35].

These network models do not aspire to produce exact replicas of complex systems; instead, they merely attempt to create network topologies that embody the fundamental characteristics of real networks. A network with one or more given characteristics can be generated by many network models, but these structural topologies are various. Therefore, except the fundamental statistical characteristics, the network structural property should receive more attention, especially for network topology design. For the fractal network model based on inverse renormalization scheme, we not only focus on the statistical characteristics of networks generated by this model, but also try to explore the distinctions of structural property of networks built by it. This fractal network model can be described as a fractal growth model, a mixing rewrite rule of node rewriting and edge rewriting. According to fractal theory, the structure of fractal unit and iterative process play an important role in fractal growth model. The primitive structure is viewed as the hybrid structure of fractal units. However, in fractal network model based inverse renormalization scheme, the effect of primitive structure on network structural property receives less attention. Therefore, in this paper, based on fractal network model by inverse renormalization, we investigate the critical effect of primitive structure on network structural property quantified by network efficiency.

2. Primitive structure divided by degree variance

Given a primitive structure with $N(0)$ nodes, a fractal scale-free network G composed of $N(t)$ nodes is generated based on inverse renormalization scheme [27] described by the following equations:

$$\begin{aligned} &N(t) = nN(t-1), \\ &k(t) = sk(t-1), \\ &D(t) = aD(t-1), \end{aligned} \tag{1}$$

where $n > 1$, $s > 1$, $a > 1$ are time-independent constants and $N(t)$ is the number of network nodes at generation t, $k(t)$ is the degree of nodes at generation t, $D(t)$ is the largest distance between nodes at generation t.

In order to investigate the critical effect of primitive structure on network efficiency, V_D, a degree variance index, is proposed to divide primitive structures in this paper. The degree variance of the primitive structure is measured by the following relations:

$$\begin{aligned} &V_D = \sum_{i=1}^{N(0)} (k_i(0) - \overline{k(0)})^2, \\ &\overline{k(0)} = \frac{1}{N(0)} \sum_{i=1}^{N(0)} k_i(0). \end{aligned} \tag{2}$$

In Fig. 1, we show six different degree sequences, S_D, for primitive structures consisted of six nodes and six edges. The degree sequences of these primitive structures are $S_{Da} = \{5, 2, 2, 1, 1, 1\}$, $S_{Db} = \{4, 3, 2, 1, 1, 1\}$, $S_{Dc} = \{3, 3, 3, 1, 1, 1\}$, $S_{Dd} = \{3, 3, 2, 2, 1, 1\}$, $S_{De} = \{3, 2, 2, 2, 2, 1\}$, $S_{Df} = \{2, 2, 2, 2, 2, 2\}$, respectively. Then, these degree variances are $V_{Da} = 12$, $V_{Db} = 8$, $V_{Dc} = 6$, $V_{Dd} = 4$, $V_{De} = 2$, $V_{Df} = 0$, respectively. It is noted that there always exist same degree variance for different topological structure, as shown in Fig. 2. In Fig. 2(a) and (b), these primitive structures have same degree sequences which are {3, 2, 2, 2, 2, 1}, {3, 3, 2, 2, 1, 1}, respectively. It is obvious that these structures with same degree sequence present different topologies. Besides, in Fig. 2(c), we display the primitive structures are the same in degree variance but different in degree sequence, {3, 3, 3, 1, 1, 1}, {4, 2, 2, 2, 1, 1}, respectively.

3. Critical effect of primitive structure on network efficiency

3.1. Theoretical analysis

For network efficiency [36,37] which was introduced to measure how efficiently information is exchanged over the network is given by

$$E(G) = \sum_{i \neq j \in G} \frac{\varepsilon_{ij}}{N(t)} N(t-1), \tag{3}$$