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Crossover transition in the fluctuation of Internet

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ABSTRACT

The inconsistent fluctuation behavior of Internet predicted by preferential attachment(PA) and Gibrat's law requires empirical investigations on the actual system. By using the interval-tunable Gibrat's law statistics, we find the actual fluctuation, characterized by the conditional standard deviation of the degree growth rate, changes with the interval length and displays a crossover transition from PA type to Gibrat's law type, which has not yet been captured by any previous models. We characterize the transition dynamics quantitatively and determine the applicative range of PA and Gibrat's law. The correlation analysis indicates the crossover transition may be attributed to the accumulative correlation between the internal links.

1. Introduction

The evolution of Internet has long been a heat subject since the dawn of complex network theory. [1–11]. With various models being continually proposed, the validation of their applicability becomes critical [1,12]. An appropriate model can strengthen not only our understanding but also our ability of control and prediction of the large-scale system.

Most popular models of Internet base on the mechanism of preferential attachment, which describes the probability of a node to capture links is proportional to its current degree. It is considered to be essential for producing the power-law degree distribution [13–17]. Some work based on mean-field approach has provided empirical evidence for its validity and derived theoretically the degree distribution exponent consistent with the reality [7,10,18–20]. Another approach to model Internet follows the Gibrat's law theory [21]. It characterizes the dynamics of the constant appearance and disappearance of links by defining the statistical properties of degree growth rate [9,22,23]. The traditional Gibrat's law indicates the growth rate is normally distributed with both its mean and standard deviation independent of the initial degree [21]. The theory succeeds in reproducing several accurate power-law exponents of Internet topology [9].

Despite the good performance of the two classical models, there exists an inconsistency between their fluctuation behavior. As is indicated in Ref [24] and will be specified in Section 2 in the present paper, the conditional standard deviation of the degree growth rate of PA decays with the initial degree as a power law of exponent -0.5, which contradicts with the assumption of Gibrat's law. Solving the contradiction requires the knowledge of the actual fluctuation of Internet. Unfortunately the mean-field method adopted by most previous work eliminates the volatility information [18–20] and few empirical studies have concerned the fluctuation of the degree growth rate in Internet. The main purpose of the present paper is to solve the issue from the view of Gibrat's law statistics, which is significant for understanding the underlying dynamics such as correlation and memory.

The paper is organized as follow. In Section 2 we show the inconsistent fluctuation between PA and Gibrat's law by deriving the conditional standard deviation of degree growth rate. In Section 3 we apply the Gibrat's law statistics to the empirical data of three different interval (daily, monthly and yearly) and find the fluctuation of Internet experiences a crossover transition with the increase of the interval. We characterize the dynamics of the transition and validate the

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applicative range of PA and Gibrat's law. In Section 4 we discuss the possible origin of the crossover transition and in Section 5 we draw the conclusion.

2. Inconsistency between Gibrat's law and PA rule

The evolution of Internet described by Gibrat's law can be formalized by the following random multiplicative process:

$$k_i(t+1) = [1+\varepsilon_i(t)]k_i(t), \tag{1}$$

where $k_i(t+1)$ and $k_i(t)$ are the degree of node i at time $t+1$ and t, and $\varepsilon_i(t)$ is a random process. The degree growth rate of unity interval is defined as

$$r_i = \log \frac{k_i(t+1)}{k_i(t)}. \tag{2}$$

Assuming $\varepsilon_i(t) \ll 1$, we can easily get the relation $r_i \sim \varepsilon_i(t)$. If we observe the system by interval Δt, the growth rate $r_i(\Delta t) = \log(k_i(t+\Delta t)/k_i(t))$ is given by

$$r_i(\Delta t) \sim \sum_{j=t}^{t+\Delta t} \varepsilon_i(j). \tag{3}$$

The property of $r_i(\Delta t)$ characterizes the dynamics of Internet's evolution. In the traditional Gibrat's law, ε_i is assumed to be (i) independent of its initial degree and (ii) uncorrelated in time [21]. According to the central-limit theorem, $r_i(\Delta t)$ is predicted to be normally distributed, and its fluctuation characterized by its standard deviation conditional to initial degree $k_0 = k_i(t)$ follows

$$\sigma_r(k_0) \sim const. \tag{4}$$

On the other hand $\sigma_r(k_0)$ of PA behaves differently. The PA rule describes the probability p_k of a new link to connect to a node relates only to the node's current degree [11], which is given by

$$p_k \propto k. \tag{5}$$

In other words the creation of links are uncorrelated with each other and the evolution of degree is a memoryless Markov process. With mean-field method, the evolution of a node's degree in a growing network follows $dk/dt = g(t)k$, where $g(t)$ is usually a function related to the growth pattern of network size. Solving the equation we have

$$k(t) \propto \frac{G(t)}{G(\tau)}, \tag{6}$$

where τ is the birth time of the node and $G(t) = e^{\int g(t)dt}$. Now Let us denote random variable $X(t)$ as the number of new links connecting to a node at time t. The identical and independent generation of each link indicates a Binomial distribution of $X(t)$, whose variance $\sigma^2_{X(t)}$ is proportional to $p_k(1-p_k)$. Considering $p_k \ll 1$, we have

$$\sigma^2_{X(t)} \sim p_k. \tag{7}$$

The degree increment of a node from t to $t+\Delta t$ is $\Delta k = k(t+\Delta t) - k(t) = \sum_{i=t}^{t+\Delta t} X(i)$. Assuming Δk is small, the definition of the growth rate naturally results in

$$r(\Delta t) \sim \frac{\sum_{i=t}^{t+\Delta t} X(i)}{k(t)}. \tag{8}$$

And the conditional variance of $r(\Delta t)$, according to the memoryless of $X(t)$, can be calculated as

$$\sigma_r^2(k(t)) \sim \frac{1}{k(t)^2} \int_t^{t+\Delta t} \sigma^2_{X(i)} di. \tag{9}$$

Substituting Eqs. (5)–(7) to Eq. (9) and replacing $k(t)$ with k_0, we finally derive the fluctuation of degree growth rate for PA

$$\sigma_r(k_0) \sim k_0^{-0.5}. \tag{10}$$

Note that Eq. (10) is valid for rewiring and link deletion as long as they do not break the memoryless property. We then arrive at a contradiction from Eqs. (4) and (10), which requires a direct look into the actual fluctuation of Internet.

On the other hand PA and Gibrat's law display common stationary property in the sense that both the scaling law of Eqs. (4) and (10) are independent of the interval Δt. This stationarity is accepted by a large number of Gibrat's law-related researches ranging from firm growth, city population, economic evolution to human dynamics [24–32]. Results obtained from a particular interval are assumed to hold true for any others and are expected to contain the full information of the underlying mechanism. However the stationarity assumption has never been examined. As will be presented in the next section, the assumption indeed breaks up and the actual fluctuation shows far richer dynamics than the existing Internet models [7–10].